A Stability Interpretation of Gödel

Anonymous

Abstract Gödel was a realist about mathematics, an idealist about time, and developed an ontological proof. Is there an epistemological principle that renders this puzzling collection of views coherent? We argue: yes, the principle of stability. This principle says that, given a model of a phenomenon, we are only justified in inferring that the phenomenon has that property if the model has this property stably, i.e., all relevantly similar models have it, too. For appropriate choices of models and similarity notions, we show that this principle precisely entails Gödel's views and thus renders them coherent. As an upshot, we assess Gödel's views by discussing these choices, and we explore alternatives. We end with three promising directions for how this investigation of stability can be done using formal methods.

1 Introduction

The 2025 Kurt Gödel Award essay question is:

> How are Gödel's conceptual and mathematical realism, his argument against the existence of time, and his ontological argument compatible with a coherent ontology?

This question alludes to the following puzzling combination of three views that Gödel held concerning ontology—i.e., the question of which objects exist.

First, Gödel believed that concepts and mathematical objects exist independently of us, which he famously expressed in the following passage:

> Classes and concepts may, however, also be conceived as real objects, namely classes as "pluralities of things," or as structures consisting of a plurality of things and concepts as the properties and relations of things existing independently of our definitions and constructions. (Gödel 1944/1990, p. 128)[1]

[1] All references to Gödel's work (including page numbers) are to the *Collected Works*. In the indicated date (e.g., 1944/1990), the first year (1944) refers to the date of the work, and the second year (1990) refers to the date of the respective volume of the Collected Works.

Second, Gödel argued that time, as we think of it, does not objectively exist. His argument is based on his construction of a universe, that is possible according to the theory of relativity, with the peculiar feature that

> in whatever way one may assume time to be lapsing, there will always exist possible observers to whose experienced lapse of time no objective lapse corresponds. (Gödel 1949/1990a, p. 205)

Third, Gödel provided an ontological argument, i.e., a proof of the existence of God. As noted by Adams (1995, p. 388), Gödel showed this proof, at the end of his life, to Dana Scott and also told Oskar Morgenstern that he was satisfied with the proof but did not publish it, as Morgenstern notes in his diary,

> [to not be misunderstood] that he actually believes in God, whereas he is only engaged in a logical investigation (that is, in showing that such a proof with classical assumptions [completeness, etc.], correspondingly axiomatized, is possible). (Quoted from Adams 1995, p. 388)

So Gödel was a realist about mathematical objects, concepts and—at least, conditional on the axioms—God, but he was an idealist about time. This collection of views is puzzling in the sense that many arguably would rather be realists about time but, say, skeptical about the Platonic existence of mathematical objects. The question hence is to explain why Gödel's views are coherent.

To start, we make the question more precise via two conceptual distinctions. First, we understand the question *systematically* and not *historically*. We do not consider—at least not very closely—how Gödel's thought developed over time leading to these three views. Rather, we ask which general principles might render these views coherent. Second, we do not understand the question *metaphysically* asking whether Gödel's three views are true. Rather, we understand the question *epistemically* asking whether the views are coherent. These two senses can come apart: We can have a collection of beliefs that are coherent—i.e., they are logically consistent, can explain the evidence we have, and adhere to scientific methodology principles like Occam's razor, etc.—but yet they are false (e.g., early modern aether theories in physics). Conversely, we can have a collection of beliefs which happen to be true, but they are not coherent (e.g., prophetically believing in a statement despite evidence to the contrary).

Hence we understand the essay question as follows: Is there an epistemological principle that renders Gödel's three views coherent? Our answer will be: Yes, the *principle of stability*. To explain, let us first state the principle and then sketch how it applies to Gödel's views—before doing this in detail in the remainder of the paper.

The principle of stability goes back to Pierre Duhem and concerns the scientific practice of studying a phenomenon by a class of mathematical models that represent this phenomenon. (See Fletcher (2020) for an excellent exposition of this principle.) Typically, the models are models of a theory about the phenomenon. For example, if the phenomenon is our universe (at a large scale), then the models are spacetimes, i.e., certain four-dimensional manifolds which are the models of the theory of relativity. But when can we infer from a given model having a property that also the phenomenon has this property? The principle of stability says that we can only make this inference if the model has this property *stably*, i.e., all sufficiently similar models also have this property. We will see that this principle is related to the axiomatic method and, as such, is also congenial to Gödel's rationalism.

Now, here is the idea how this applies to Gödel's views. Each view concerns one phenomenon: namely, (a) mathematics, (b) time, and (c) theism. Each phenomenon is studied via the models of the best available theory for the respective phenomenon: namely, (a) ZFC, i.e., Zermelo–Fraenkel set theory with the axiom of choice, (b) Einstein's general theory of relativity, and (c) Gödel's axiomatization of God as an entity that has all positive properties. Concerning (a), absoluteness results in ZFC ensure that all models agree on, e.g., the natural numbers and many other mathematical objects. So the property that the natural numbers exist is stable, and hence can be inferred—or so the argument goes—to also hold of the phenomenon, just like Gödel's view claims. Concerning (b), in the spacetime that Gödel constructed, time is not as we think it is, so—or so the argument goes—the existence of an objective lapse of time is not stable, hence we cannot infer it to hold of the phenomenon, like Gödel claims. Concerning (c), Gödel's proof shows that God, i.e., an entity that has all positive properties, exists, and hence this holds in all models of Gödel's theory. So the property that God exists is stable, and hence can be inferred—or so the argument goes—to also hold of the phenomenon.

Thus, the principle of stability renders Gödel's three views coherent. In this paper, we develop and discuss this answer to the essay question. The structure is as follows. In Section 2, we describe the just mentioned model-based conception of science and the principle of stability. In Section 3, we go through Gödel's three views and show how it is justified by the principle of stability. In Section 4, we discuss three ways to change a scientific modeling context: by adding more principles to the theory, by considering other properties to be inferred about the phenomenon, and by changing the underlying notion of similarity. In Section 5, we sketch three avenues for an investigation of stability with formal methods. We conclude in section 6.

2 Scientific Modeling Contexts and the Principle of Stability

In this section, we describe the model-based conception of science and state the principle of stability. We also compare it with the axiomatic method and Gödel's rationalism.

For the purpose of this paper, we assume the following caricature conception of science.[2] Each scientific endeavor aims to understand some *phenomenon*. Typically, this endeavor is made more precise by asking whether the phenomenon has a certain *property* that the scientists are interested in. An example already mentioned is general relativity: Here, the phenomenon is our universe at large, and one property of interest is whether the universe has a 'global time', i.e., a lapse of time that all observers agree on. (We discuss this in detail in Section 3.1.) To answer this question, scientists develop a *theory* and corresponding *models* of this phenomenon. The theory can express the property and describes the laws that govern the phenomenon. The models of the theory represent the possible ways the phenomenon could be, given these laws.[3] Hence, the models are *epistemically possible worlds*: the ways the phenomenon could be given what we know about it. One of those worlds represents the actual phenomenon, but we do not know which. Gathering new evidence will make some worlds more likely than others, but all of them are possibilities. In the general relativity example, the theory is Einstein's general theory of relativity, and the models are spacetimes, i.e., certain four-dimensional manifolds.

The principle of stability concerns when we can infer that the phenomenon has the property from a model having the property. It says that this is only possible if the model has the property stably, i.e., all relevantly similar models have the property, too. Hence, to state the principle, we also need a notion of similarity between models.

To give a name to this bundle of ideas, let us say that a *scientific modeling context* consists of a phenomenon, a property of the phenomenon, a theory of the phenomenon, a class of models of the theory, and a notion of similarity between the models.[4] In such a context, we can state the principle of stability (Fletcher 2020).

Principle of stability (necessity) The inference from a model having property φ to the phenomenon having property φ is justified only if the model has property φ stably, i.e., all relevantly similar models also have φ.

To illustrate, say our scientists have some convincing evidence that a particular spacetime M accurately represents the actual universe. They mathematically prove that M has the property φ of having a global time. Are they justified to conclude that the universe has this property φ? Well, the evidence

[2] By 'caricature' we want to stress that we do not want to say that all science is like this or should be like this, but that this conception rather is a useful and idealized picture of science that covers many instances.

[3] We do not further distinguish the phenomenon type and the tokens of it. For some phenomena (e.g., our universe), there is just one token in the actual world, but other phenomena are multiply realized in the actual world (e.g., population dynamics).

[4] The word 'context' is to reflect the 'methodological contextualism' suggested by Fletcher (2016).

they gathered is subject to measurement errors and other imprecision, so it does not single out M uniquely. Models similar to M are also consistent with that evidence, and if one of them violates φ, we cannot consistently with the evidence conclude φ about the universe. In other words, only if M and all similar models have property φ, are we justified to conclude φ about the universe.

As stated, the principle only restricts inferences: it requires stability as a necessary condition for justified inference ('only if'). However, in practice, we are also interested in sufficient conditions, i.e., how much stability is needed to justify an inference ('if' rather than 'only if'). One such sufficient amount of stability is maximal stability:

Principle of stability (sufficiency) If all models have property φ ('maximal stability'), then the inference from a (or any) model having property φ to the phenomenon having property φ is justified.[5]

We take the 'principle of stability' to include both the necessity and the sufficiency versions. In the remainder of this section, we add five clarifying comments.

(1) *Static vs dynamic contexts*. For now, we work *statically* in a fixed scientific modeling context. The scientists are in the business of understanding the space of models, gathering evidence which models are more likely to be actual, evaluating the stability of the property of interest, etc. Later, in Section 4, we will also *dynamically* consider changing context. For example, the scientist might have gathered overwhelming evidence to add a new law to the theory (and thus declare models that do not satisfy it as not epistemically possible after all). Or maybe the scientists discard the theory completely in favor of a new one, etc. Maybe the scientists are also led to consider a related property which, however, is not yet expressible in the theory, so it also requires a revision of the theory.

(2) *Realism*. This conception of a model-based science comes naturally with realism, i.e., the assumption that there is some reality that generates the phenomenon. In the general relativity example, this would be the assumption that the phenomenon, i.e., our observations of the stars, originates from a real world that exists independently of us and that contains these stars. Here, however, we remain neutral on this: We do not make any metaphysical assumption about the origins of the phenomena. We are only concerned with the epistemic question which inferences about the phenomenon are justified by the theory and its models, and we are not concerned with the metaphysical question whether these inferences are true.

[5] Fletcher (2020, p. 13) discusses this as part of a principle dubbed 'Justification' which is argued to imply, together with two other principle, the above principle of stability: "If several models represent the phenomena equally well, then only those properties which all those models have are licensed [to have justification to infer of phenomena]".

(3) *Justification of the principle*. The principle has been suggested with the natural sciences in mind, although, as Fletcher (2020, p. 2) notes, it "appears to be of unusually wide scope, applying in principle to inferences from properties of models in any science when one can describe those models with sufficient precision". The justification for the principle is a 'contingent transcendental argument': If we engage in the scientific modeling of a phenomenon as described above (and it is a contingent matter whether we do), then we must accept this principle (hence it is transcendental, i.e., constitutive of the activity of scientific modeling). See Fletcher (2020, sec. 5) for an extended argument, with a reference to Chang (2008) discussing the idea of a contingent transcendental argument. Also see the literature on robustness analysis in science for related ideas: Levins (1966), Weisberg (2006), Woodward (2006), and Schupbach (2018)—see Fletcher (2020, App. A) for a comparison with the principle of stability.

(4) *Axiomatic method*. The principle of stability is reminiscent of the axiomatic method. The latter also postulates basic principles about the phenomenon, and insights into the phenomenon are gained by deducing theorems from these axioms. This method, of course, has been extremely influential not just in mathematics (Hintikka 2009), but also in economics (Thomson 2001) and the physical and social sciences (Suppes 1983). Just as the axiomatic method justifies consequences of the axioms, the sufficiency version of the principle of stability also justifies inferring those properties that all models of the theory have, i.e., those properties that logically follow from the theory. And if similarity is trivial, i.e., every model is similar to any model, then the necessity version of the principle of stability says that a property that does not follow from the axioms, i.e., is not stable (since false in some model of the theory), also cannot justifiably be inferred about the phenomenon. However, what the principle of stability adds over the axiomatic method, is a more fine-grained analysis of the space of models of a theory, by taking similarities into account and not just logical consequence.

(5) *Gödel's rationalism*. The axiomatic method is also congenial to Gödel's rationalism, who was optimistic about finding, via conceptual analysis, basic axioms from which not just mathematical but also philosophical questions can be decided. For a discussion of this, see Kennedy (2020, sec. 3) and van Atten and Kennedy (2003).

3 The Coherence of Gödel's Views: Stability of Concepts

In this section, we argue that the principle of stability renders Gödel's three views coherent. In general, these arguments take the following form, starting from a given phenomenon and a property that we would like to infer about the phenomenon.

(P1) Identify the scientific modeling context, i.e., (a) the theory of the phenomenon and the corresponding class of models, and (b) the notion of model-similarity.

(P2) Prove a mathematical result about the non-stability (resp., maximal stability) of the property φ among the class of models.

(C) Conclude with the stability principle that φ cannot (resp., can) be justifiably inferred about the phenomenon.

In the next three subsections, we do this for Gödel's three views. For the phenomenon of time (Section 3.1), we provide a scientific modeling context in which the principle of stability entails that there is no justification for inferring the property that an objective time exists. For the phenomenon of mathematics (Section 3.2), we provide a scientific modeling context in which the principle of stability entails that there is justification for inferring the property that various mathematical objects exist. For the phenomenon of theism (Section 3.3), we provide a scientific modeling context in which the principle of stability entails that there is justification for inferring the property that God exists. In each case, we also critically discuss the suitability of the chosen scientific modeling context.

3.1 Idealism about Time

Gödel (1949/1990a) criticized our conception of time according to which any two events are objectively (and not just observer-relatively) ordered as either simultaneous or one before the other. Gödel described this view thus: "reality consists of an infinity of layers of 'now' which come into existence successively" (Gödel 1949/1990a, 202f.). His argument involved two steps. First, Gödel (1949/1990b, 1952/1990) proved that there is a spacetime—i.e., a model of the theory of general relativity—that cannot have a global time: there is no way of ordering the events of the spacetime into layers of now. This (type of) spacetime is now known as the *Gödel universe*. Second, Gödel (1949/1990a) claimed that this possibility renders our conception of time philosophically unsatisfactory:

> It might, however, be asked: Of what use is it if such conditions prevail in certain *possible* worlds? Does that mean anything for the question interesting us whether in *our* world there exists an objective lapse of time? I think it does. ... For, if someone asserts that this absolute time is lapsing, he accepts as a consequence that whether or not an objective lapse of time exists (i.e., whether or not a time in the ordinary sense of the word exists) depends on the particular way in which matter and its motion are arranged in the world. This is not a straightforward contradiction; nevertheless, a philosophical view leading to such consequences can hardly be considered as satisfactory. (Gödel 1949/1990a, 206f.)

In this subsection, we interpret Gödel's argument as an argument from the principle of stability. Then we discuss its plausibility: after all, while the first step has been celebrated as an important insight into the theory of general relativity, the second, modal step has been heavily criticized (e.g. Earman 1995, pp. 194–200).

So we need to spell out premise (P1) and premise (P2) of argument pattern, where the phenomenon and property under discussion are our universe at large and whether it has a global time.

Regarding premise (P1), part (a) is clear: Our best available physical theory about the universe at large is the theory of general relativity. Its models, called spacetimes, are pairs (M, g), where M is a connected four-dimensional Hausdorff C^∞ manifold and g is a Lorentz metric such that (M, g) is an exact solution of Einstein's field equations (Hawking and Ellis 1973, p. 56 and p. 117). We do not need the formal definition in the remainder, so we just elaborate on the intuition: The manifold M contains the points—known as *events*—of spacetime (with three spatial dimensions and one temporal dimension), g describes the geometry, the field equations interlink the geometry and the matter, and exactness requires a 'reasonable' distribution of matter.

Having specified the theory and its class of models, only part (b) remains: the notion of model-similarity. As the preceding quote shows, Gödel took the spacetime that he constructed to be relevant when considering the property of having a global time also in the actual spacetime. This suggests the trivial notion of similarity: any spacetime is relevantly similar to the given one when considering the property of having a global time. This is what we choose for now as our interpretation of Gödel's argument. But later, when we discuss the plausibility of the argument, we also consider other notions of similarity.

Regarding premise (P2), Gödel's universe proves the non-stability of the property of having a global time: Whatever the actual spacetime, under the chosen trivial similarity, there always is a relevantly similar spacetime—namely, Gödel's universe—that does not have a global time.[6]

Now, the principle of stability implies (C): we cannot justifiably infer about the actual universe that it has a global time. This is how we interpret Gödel's conclusion: that the view that "time in the ordinary sense of the word exists" is unsatisfactory, i.e., not justified.

Thus, in short, our interpretation of Gödel's argument is this. Premise (P1) states the implicit background assumptions of Gödel's argument. Premise (P2) is the first step of Gödel's argument: the existence of the Gödel universe. The principle of stability is the second, modal step of Gödel's argument. And (C) is Gödel's conclusion against the existence of time.

The upshot of this interpretation of Gödel's argument is that its plausibility

[6]Formally, a spacetime $(M$ has a global (or cosmic) time if there is a function $t : M \to \mathbb{R}$ such that, for all events $p, q \in M$, if p causally precedes q (so some signal can be send from p to q), then $f(p) \leqslant f(q)$.

now hinges on the plausibility of the choice of model-similarity. In particular, it prompts us to ask why we should consider trivial similarity and not another, more careful notion. Here are some intuitive examples of such alternative notions from the literature. A spacetime (M', g') is similar to a given spacetime (M, g) if ...

1. *Observational similarity*: the parameters determining (M', g') are within the error margin of measuring these parameters in (M, g).[7]
2. *Uncertainty similarity*: the difference in the parameters determining (M', g') and (M, g) are beyond what is detectable according to the uncertainty principle.[8]
3. *Civilizational similarity*: an arbitrarily advanced civilization of (M, g) could locally manipulate matter and energy to obtain (M', g').[9]
4. *Indistinguishability similarity*: if we were to live in (M', g'), we would experience time as we do in (M, g).[10]

These notions of similarity have some intuitive motivation, so the question is, if we adopt them, instead of trivial similarity, would the argument from the principle of stability still go through? To answer this, we would need to make the intuitive notion of similarity mathematically precise, in order to prove the non-stability as a mathematical fact. We will take up the question of formalization in Section 4.3 using topology, and for discussions of the informal notions we restrict ourselves to some pointers to the literature.

Regarding observational similarity, there is some discussion of whether the Gödel universe is consistent with the measurement of the actual universe. In the standardly considered cosmological models, the Friedmann-Lemaître-Robertson-Walker spacetimes, there is a global time (e.g. Smeenk and Wüthrich 2011, sec. 4). However, one may regard Gödel (1952/1990) as making his argument more convincing by constructing universes that have no global time and are observationally more plausible. Indeed, Hawking (1990, 189f.) writes that "[t]hese models could well be a reasonable description of the universe that we observe, although observations of the isotropy of the microwave background indicate that the rate of rotation must be very low". For further discussion, see Su and Chu (2009). Quantum similarity is difficult

[7]This is arguably the most straightforward idea. To provide but one reference, see Hawking (1971, p. 395).

[8]This is suggested by Hawking and Ellis (1973, p. 197): "General Relativity is presumably the classical limit of some, as yet unknown, quantum theory of space–time and in such a theory the Uncertainty Principle would prevent the metric from having an exact value at every point. Thus in order to be physically significant, a property of space–time ought to have some form of stability".

[9]See, e.g., Hawking (1992) or Smeenk and Wüthrich (2011).

[10]See, e.g., Savitt (1994), Yourgrau (1999, p. 47), Gödel (1949/1990a, p. 206), or Smeenk and Wüthrich (2011). Another way to formulate this is: our cognition of time tracks the physical time. (This is reminiscent of the characterization of knowledge of Nozick (1981) according to which, roughly, a belief is knowledge if it tracks the truth.)

to assess given the open problem of merging the theories of quantum mechanics and general relativity. Regarding civilizational similarity, whether a spacetime without a global time can be rendered actual depends, arguably, on the much-discussed cosmic censorship and chronology protection conjecture. For indistinguishability similarity, the argument would be that, for any local region of spacetime (say, a human lifetime), the experience cannot distinguish between a Gödelian rotating universe and a non-rotating universe with a global time, if the rotation is very low. For discussion, see, e.g., Savitt (1994), Belot (2005), Dorato (2002), and Smeenk and Wüthrich (2011).

3.2 Realism About (Parts of) Mathematics

Next, we move to mathematics, a domain for which the principle of stability was arguably not directly envisaged. But given the similarity to the axiomatic method and given the "unusually wide scope, applying [whenever] one can describe those models with sufficient precision" (both noted in Section 2), we explore the application nonetheless. We ask: can the principle of stability be invoked to convey justification to Gödel's view that, at least certain, mathematical objects exist?

Before we start, though, recall that our goal is *not* to argue that Gödel's Platonistic beliefs are true. In other words, we do not aim to produce another argument in the philosophy of mathematics for Platonism (e.g., a classic such argument is the Quine–Putnam indispensability argument). Instead, the question we are concerned with here is how the principle of stability can convey *some* justification for Gödel's position—regardless of whether this justification ultimately holds up scrutiny or whether Platonism is true. So we need to provide a scientific modeling context (P1) and show the maximal stability of the existence of mathematical objects (P2). Then the principle of stability renders Gödel's views coherent, as we aim to show qua answer to the essay question. Once this is in place, future work can discuss, e.g., whether Gödel would have put forward this justification himself or whether the justification holds up scrutiny and can be taken as an argument for Platonism.[11]

Regarding premise (P1), what is an appropriate scientific modeling context for mathematics? Despite the somewhat odd phrasing, this question has a clear answer. The axiomatic foundations—and hence the theory—of all of mathematics is ZFC, i.e., Zermelo–Fraenkel set theory with the axiom of choice. The models of this theory are those in the sense of model theory, i.e., structures (M, E). Here M is a set (namely the set of objects the model considers to be sets) and E is a binary relation on M (where xEy means that x is an element of y) such that, for all axioms $\psi \in \mathsf{ZFC}$, the model (M, E) satisfies ψ, written $(M, E) \models \psi$. This settles part (a) of (P1); and regarding part (b), we choose the trivial notion of similarity, where every model is similar to every other model,

[11] For discussion of Gödel's platonism, see, e.g., Hintikka (1998) and van Atten (2001).

because we want to establish maximal stability.

Regarding premise (P2), we need to show that the property of 'x exists', for different mathematical objects x, is maximally stable across the models of ZFC. For concreteness, let us do this for x being the set $\mathbb{N}$ of natural numbers.

We need to be a bit more precise about what we mean by this existence claim. In a scientific modeling context, we consider a class X of models of a theory T. These models are mathematical objects, so to define them and to prove properties about them, we work in some mathematical background theory, call it B. Usually, B is not even mentioned explicitly—for instance, we did not specify it in Section 3.1—and just taken to be ZFC, i.e., all of mathematics. (Often just a fragment of ZFC is enough, but we can be permissive here.) Within this background theory B, we can consider the mathematical objects that are models of the theory T. For instance, in Section 3.1, T was the theory of general relativity, and the models were certain four-dimensional manifolds. Now, however, T is ZFC and the models are model-theoretic structures (M, E). Isn't this circular? No, in our background theory $B = \mathsf{ZFC}$, we can do all of mathematics including differential geometry (studying manifolds) and model theory. The only added subtlety is that we need to distinguish between what we, in our background theory, consider to be a set, and what the structure (M, E) considers to be a set. To be more precise, in set theory, one writes V for the collection (the 'class') of all objects of our background theory. The structure (M, E) is one such object, so (M, E) is in V. Moreover, (M, E) is built out of objects of our background theory, so each element of the set M also is an element of V, so $M \subseteq V$. However, not every set in V needs to be a set in M, i.e., $V \not\subseteq M$.[12]

Now, we can make precise the existence claim. In our mathematical background theory B, we acknowledge, as anywhere in mathematics, the set $\mathbb{N}$ of natural numbers. So the question whether the object $x = \mathbb{N}$ exists in all models of ZFC becomes the question whether the set $\mathbb{N}$ is in M for all models (M, E) of ZFC. That this is the case is precisely the content of *absoluteness theorems* in set theory.[13] To state them, we need to more ideas. First, set theorists are typically only interested in models (M, E) that, like V above, also have the property that, if $x \in M$, also $x \subseteq M$ (i.e., the elementhood relation E is transitive). These are known as *transitive* models of ZFC. Second, we can uniquely define the natural numbers by the property $\varphi_{\mathbb{N}}(x)$ saying that x is the smallest set that contains 0 (which we identify with the empty set) and is closed under the successor operation (which we define as $S(y) := y \cup \{y\}$).[14] In other words, our background theory ZFC proves that there is a unique object x with the property $\varphi_{\mathbb{N}}(x)$, and we denote this object by $\mathbb{N}$. Now, if (M, E) is a model

[12] Since M is a set, it has a powerset, which is in V, but cannot be in M.

[13] See, e.g., Kunen (1980, ch. IV) or Jech (2003, ch. 12).

[14] Formally, we define $\varphi_{\mathbb{N}}(x)$ as saying that x is a limit ordinal, x is not 0, and every member of x is a natural number (i.e., an ordinal that, if not 0, is not a limit ordinal, and every element of it also has this property). See the proof of Jech (2003, lem. 12.10).

of ZFC, it also has a unique object x_M such that $(M, E) \models \varphi_{\mathbb{N}}(x_M)$. But the question is this: is this object x_M in M also what we consider to be the natural numbers, i.e., $x_M = \mathbb{N}$? If (M, E) is transitive, this is what the absoluteness theorem tells us: For all $y \in M$, we have

$$(M, V) \models \varphi_{\mathbb{N}}(y) \Leftrightarrow \varphi_{\mathbb{N}}(y), \tag{1}$$

so, setting $y := x_M$, we get that the unique object x_M in M that satisfies $\varphi_{\mathbb{N}}(y)$ in M is also the unique object in V that satisfies $\varphi_{\mathbb{N}}(y)$, hence $x_M = \mathbb{N}$.

Thus, all models of ZFC agree that the set of natural numbers, as we as modelers think of it, really exists. So the principle of stability justifies inferring this existence about the phenomenon. Hence this is in contrast to Section 3.1, where the notion of time, as we as modelers think of it (namely, as objective layers of 'now'), did not exist in all models, so the principle of stability precluded a justified inference to the phenomenon. Again, we do not claim that Gödel made—or would have made—such an argument, however Gödel defined the notion of absoluteness in his work on the axiom of choice and the continuum hypothesis (Gödel 1940/1990, p. 76).[15]

In this way, the principle of stability justifies the existence of *one* mathematical object, albeit an important one, namely the set of natural numbers. But of course, Gödel's view included the existence of *all* mathematical objects. So can we push this argument further? The absoluteness theorems also apply to many other notions: like being a pair, being a function, being an ordinal, being a Cartesian product, etc. So we can similar justify other mathematical objects and concepts. However, there are also bounds to this: absoluteness can fail. So the principle of stability only justifies the existence of some but not all mathematical objects.

To understand failures of absoluteness, note that the absoluteness argument involved two steps. First, there is a formula that uniquely defines the object of interest ($\varphi_{\mathbb{N}}(x)$ above), and second we show that the interpretation of this formula is 'the same' in the different models (equation 1 above). So the argument can fail because either (1) there is no uniquely defining formula for the object or (2) there is such a formula, but it changes its meaning across models.

An example of (2) is provided by Cohen's famous proof of the independence of the continuum hypothesis (completing the work started by Gödel). One can define the real numbers—also known as the continuum—as $2^{\mathbb{N}}$, but what that object is in different models varies: in some models it has the cardinality $\aleph_1$ (so the continuum hypothesis is true there) and in others it has the cardinality $\aleph_2$ or yet other cardinalities (so the continuum hypothesis is false there). Hence the continuum is not an absolute object and its existence cannot be justified with the principle of stability.

[15]See Solovay (1990) for discussion.

An example of (1) is V, and Gödel was very excited about that. In a talk, Gerald Sacks recounts a dinner with Gödel, Morgenstern and others once at the Institute for Advanced Study in Princeton:[16]

> [Gödel] brought up … one of his favorite concepts, the Absolute, with a capital 'A'. That's not a concept that 20th century philosophers tend to bring up: the Absolute. So what was it to him? In my mind, it was the class of all sets, but maybe it was something more for him, I'm not sure. But he seemed to be talking about the class of all sets, hence of all mathematical objects; that's worthy of being called the Absolute. … He says: 'You know, language does not enable us to define the Absolute'. If you have a formula, $F(x)$, it can never happen that that formula has exactly one x that satisfies it, that x being the Absolute, that can't happen. Because we know from the reflection principle in set theory, if I have a formula $F(x)$, and $F(V)$ holds where V is the class of all sets, then F must also hold for some set. In other words, anything you say about the class of all sets which is true, is also true for some particular set. And hence you can't define the Absolute. … Then his eyes lit up, he said: 'Isn't that wonderful? We know something important about the Absolute, just by logic, just by reason'. And he was full of enthusiasm.

In sum, when applied to mathematics, the principle of stability essentially amounts to the axiomatic method and justifies the existence of objects that are absolute in ZFC. We will take up the question of whether this can be extended to more objects in Section 4.1.

3.3 Ontological Argument

Finally, we move to the third view: Gödel's ontological argument. As mentioned in the Introduction, the ontological proof of Gödel (1970/1995) was never published, but Gödel worked on it at least since 1941 and told Dana Scott in 1970 about it. Scott presented a version of the proof in his seminar, so it became fairly well-known. For a history and broader context of Gödel's proof, see, e.g., Adams (1995), Benzmüller and Scott (2025), and Sobel (2004).

Here, we want to interpret Gödel's ontological argument in terms of the principle of stability. So we again need to spell out premises (P1) and (P2).

Regarding premise (P1), part (a) now of course cannot rely on an established theory of theistic phenomena. Instead, Gödel postulates such a theory with the axioms of the proof. It defines an object to be God iff it has all positive properties; and it provides axioms on the notion of a positive property. These

[16] For a recording of the talk, see `https://www.youtube.com/watch?v=PR7MTqtFl4Y`, starting at minute 25:39 (last checked on 22 June 2025).

axioms include, for example, that any property either is positive or its negation is positive (and not both); or that if a property is positive, it necessarily is positive. These axioms are formulated in a higher-order modal logic, so the models of Gödel's theory are the models of higher-order modal logic satisfying the axioms of the theory. (For details, see Benzmüller and Woltzenlogel Paleo (2014).) For part (b), we again choose the trivial notion of similarity, because we want to establish maximal stability.

Regarding premise (P2), this now is Göedel's proof: that the axioms logically imply the necessary existence of God, i.e., an object that has all positive properties. Hence the existence of God holds in all models of the theory and thus is maximally stable. Scott's version of Gödel's proof was formally verified using Automated Theorem Provers by Benzmüller and Woltzenlogel Paleo (2014). For a recent discussion and many further references, see Benzmüller and Scott (2025).

Hence, in this scientific modeling context, the principle of stability justifies inferring the existence of God—again with close affinities to the axiomatic method. Our goal is to show how the principle of stability renders Gödel's ontology coherent. For a further discussion of Gödel's argument itself, we refer to Sobel (2004).

4 Dynamics: Changing the Scientific Modeling Context

So far, we have applied the principle of stability in a fixed scientific modeling context. Moving from this static setting to a dynamic setting, we now want to consider what happens when this context changes. We consider three possible such changes: we change the theory by adding further principles (Section 4.1), we change to a different property under discussion (Section 4.2), or we change the notion of similarity (Section 4.3).

4.1 Changing Theory: Toward New Axioms of Set Theory

We saw in Section 3.2 that the principle of stability only justified the existence of some but not all mathematical objects. It justified all those objects that are absolute. However, because of the independence of the continuum hypothesis, objects related to the continuum are notoriously not absolute. Thus, a natural question—at least from a Platonist's view—is whether the theory, i.e., ZFC, can be plausibly extended to settle the continuum hypothesis and to, hopefully, thus also justify more objects. Indeed, Gödel was programmatic in initiating a search for new axioms in set theory:

> one may on good reason suspect that the role of the continuum problem in set theory will be this, that it will finally lead to the discovery of new axioms which will make it possible to disprove Cantor's conjecture. (Gödel 1947/1990, p. 196)

To this day, this continues to inspire work in set theory, including the recent celebrated result of Asperó and Schindler (2021), who also cite the above passage. They unify two prominent but conceptually different axiom candidates that both settle the continuum hypothesis in the negative: namely, confirming the belief of Gödel, which he held at least for parts of his life (Moore 1990, sec. 7), that the continuum in fact has size $\aleph_2$.

This arguably is the most famous instance of Gödel discussing the search for new axioms. But we may also ask this for the other applications of the principle of stability that we have discussed. For example, Kahle (2023) argues this to be the take-away from Gödel's argument about time: that we have to look for new physical principles that exclude the Gödel universe.

4.2 Changing Property: Modal Arguments in General Relativity

In this subsection, we consider examples in general relativity of changing the property under discussion. We consider examples that leave the rest of the scientific modeling context unchanged. In other words, we do not consider examples for which we need to make the theory more expressive.[17]

One example of such a different property is that the universe is physically deterministic, i.e., any future or past physical state of the universe is determined by some (e.g., the current) state together with the physical laws. One way to make this property of the universe precise is to demand that our universe has a Cauchy surface: a surface of spacetime such that any given initial conditions on this surface determine the future and past. Having a Cauchy surface is equivalent to being globally hyperbolic, which implies being stably causal, which, in turn, is equivalent to having a global time. Thus, Gödel's argument also applies to the property of being physically deterministic: in the context of general relativity, it is not stable with respect to the trivial similarity, so we are not justified to infer it about the universe. Discussions of the plausibility of the original argument hence apply similarly to discussion of the plausibility of the present argument. What we want to stress, though, is that thus Gödel's argument generalizes—via the principle of stability—to a *modal argument pattern* in general relativity.

Indeed, we now argue that we can regard another extensive debate in the philosophy of general relativity theory as an instance of this modal argument pattern. This debate concerns the validity of the Church–Turing thesis when moving from our everyday notion of space and time to spacetime as described by general relativity.

This started when, based on an idea of Pitowsky (1990), Hogarth (1992) showed that there are spacetimes with the following startling property. There are two idealized observers, Alice and Bob, who travel through spacetime

[17] We would need to make the theory more expressive, for instance, to formalize the indistinguishability notion of similiarity, since it involves a subject's time experience in a spacetime.

heading off in two different directions. Bob's journey is indefinitely long: his clock will keep running forever. However, after some finite time on Alice's clock, all of Bob's infinitely long journey is in the accessible past of Alice. So for Alice "forever is a day" (Earman and Norton 1993). This event on Alice's worldline is called a *Malament-Hogarth event* (*MH-event*) and spacetimes where this is possible are called *Malament-Hogarth spacetimes* (*MH-spacetimes*).

The interest in MH-spacetimes comes from the fact that they seem to allow for hypercomputation: for example, Alice and Bob can conspire to 'compute' the non-Turing computable problem of whether or not the axioms of set theory, ZFC, are consistent. Bob goes through every ZFC-proof and checks if it is a proof of the inconsistent sentence $\perp$. If he finds a proof, he sends a light signal to Alice which she will have received at the MH-event. So if Alice receives a signal, she knows, after some finite time (on her clock), that ZFC is inconsistent, and if she does not receive a message by then, she knows that ZFC is consistent. In fact, Welch (2008) answered the question of how much more exactly can be computed in MH-spacetimes. By appropriately 'stacking' observers like Alice and Bob, it is shown that all hyperarithmetical predicates can be computed in some MH-spacetime.[18]

MH-spacetimes generated much discussion: are they 'physically reasonable' or are they merely possible according to general relativity but violate quantum mechanical or other physical laws? See, e.g., Etesi and Németi (2002), Németi and Dávid (2006), Manchak (2010), Andréka et al. (2018), or Bournez and Pouly (2021, sec. 6.2.9). Here, as mentioned, we note that this debate can be seen as an instance of the modal argument pattern. The phenomenon still is the universe, but the property under discussion is now the impossibility of hypercomputation or, equivalently, the correctness of the physical Church–Turing thesis (Andréka et al. 2018). The scientific modeling context is still that of general relativity. But the upshot is that the debate can now be framed as being about the choice of similarity notion and the corresponding stability or instability of the property.[19] The question is: Are there principled notions of similarity that render the impossibility of hypercomputation stable and hence justified? Or do natural notions of similarity include MH-spacetime that render the physical Church–Turing Thesis unstable and hence unjustified? We now turn to formalizing such questions of stability.

4.3 Changing Similarity: Topologies on the Space of Models

If X is a class of models, a natural way to formalize a notion of (graded) similarity—and thus stability—is via topology. This is described in detail by

[18] In the rotating black hole MH-spacetimes considered by Etesi and Németi (2002), this reduces to (a subclass of) Δ^0_2-predicates.

[19] There is a potential connection with the above discussion of physical determinism, since Etesi (2002, 2013) have argued that the Church-Turing thesis is intimately related to the cosmic censorship hypothesis (which is closely connected to determinism).

Fletcher (2020, sec. 4) and Fletcher (2018b).[20] The idea goes as follows.

Given a model x in X, we represent a degree of similarity to x by the set U of models that are similar to x to that degree. Let us write $\mathcal{N}_x$ for the collection of x's similarity degrees. The following assumptions then are natural:

- For all $U \in \mathcal{N}_x$, we have $x \in U$ (i.e., x is always similar to itself).
- For all $U, U' \in \mathcal{N}_x$, there is $V \in \mathcal{N}_x$ such that $V \subseteq U \cap U'$ (i.e., being similar to x both to degree U and to degree U' is implied by some degree of similarity to x).
- For all $U \in \mathcal{N}_x$ and $y \in U$, there is $V \in \mathcal{N}_y$ such that $V \subseteq U$ (i.e., if y is similar to x, then objects sufficiently similar to y are also similar to x).

This determines a *topology* on X in the usual sense, i.e., a collection τ of subsets of X that is closed under finite intersection and arbitrary union (Engelking 1989, ch. 1).[21] Conversely, if τ is a topology on X, we can define, for $x \in X$, the collection $\mathcal{N}_x := \{U \in \tau : x \in U\}$ of neighborhoods of x.

In sum, a notion of similarity on the class of models M is described by a topology τ on X. The members of τ—known as *open sets*—describe degrees of similarity: if two models x and y are in an open set U, this means that they are similar to degree U; the smaller U, the closer the similarity.

Within this topological framework, we can explicate further concepts. First, we can identify properties of models with subsets P of X: model x having property P means $x \in P$. We can then also explicate the idea that model x has property P stably, i.e., all relevantly similar models y also have the property:

$$\text{there is } U \in \tau \text{ such that } x \in U \text{ and, for all } y \in U, \text{ we have } y \in P.$$

We call a property $P \subseteq X$ *stable* if any model $x \in X$ that has P has it stably. It is not difficult to show that, thus, the stable properties are precisely the open subsets.[22]

Second, we can also formalize the argument from the principle of stability: Given a phenomenon and a property P of it, first identify (1)(a) the theory of the phenomenon and its class of models and (1)(b) specify a topology on the class of models. Next (2) prove one of the following:

(i) Stability: the property P is generic, i.e., open and dense in the topology.[23] This is the topological analogue of the probabilistic idea that the property

[20] Fletcher and Lackey (2022) provides a broader historical context on the use of topological ideas in philosophy.

[21] Namely, τ contains those subsets U of X that are unions of subfamilies of $\bigcup_{x \in X} \mathcal{N}_x$ (Engelking 1989, prop. 1.2.3).

[22] Since the collection of all open sets forms a Heyting algebra, the logic of stable properties hence is intuitionistic logic. See Vickers (1989) for a more general verificationalist (or, dually, falsificationist) interpretation of topology.

[23] A subset P of a topological space (X, τ) is *dense* if, for every $x \in X$ and for every open set $U \in \tau$ with $x \in U$, we have $P \cap U \neq \emptyset$.

holds for 'almost all' models.[24] Thus, this generalizes the sufficient condition of the principle of stability.

(ii) Non-stability: the property P is not stable (or, stronger, no model has P stably, i.e., $\neg P$ is dense). Thus, this formalizes a violation of the necessary condition of the principle of stability.

Then conclude with the principle of stability that P can (if (i) holds) or cannot (if (ii) holds) be justifiably inferred about the phenomenon.

Now, since notions of similarity correspond to topologies on X, the question about 'the right' notion of similarity turns into the question of 'the right' topology on X. Let us discuss this question for the specific example of general relativity. The 1970s saw extensive research into what is the best topology on the class of spacetimes.[25] Two topologies have been suggested as the main contenders: the *open topology* and the *compact-open topology*. We do not need the formal definitions here, but the high-level idea is this. The compact-open topology considers two metrics g and g' on a manifold M to be similar if they have similar values on a compact region of spacetime (the bigger the region, the more they are similar), while the open topology requires this agreement in all of spacetime.[26]

What does this mean for Gödel's argument for the ideality of time, in the above formalized version? On the one hand, in the open topology, the theorem of Hawking (1969)—that having a global time is equivalent to stable causality—implies that having a global time is a stable property. Hence, it is generic that either a spacetime contains closed timelike curves (i.e., blatant violations of a global time) or it has a global time (Hawking 1971, p. 397). Hence we can justifiably infer this disjunction for our universe, and Gödel's argument for the ideality of time does not succeed. On the other hand, in the compact-open topology, failing to have a global time is dense (Fletcher 2016, prop. 4, p. 376). Hence we cannot justifiably infer that the universe has a global time, and Gödel's argument does succeed.

However, Geroch (1970, 1971) criticizes both the open and the compact-open topologies as not quite adequate for capturing our intuitions of similarity. Fletcher (2018a) analyzes this criticism into an impossibility result and also provides, for slightly weaker demands, a possibility result. This introduces a new topology, called the global topology, in which the counterpart of Hawking's theorem still holds.

[24]See Oxtoby (1980) for analogies and disanalogies between topological and measure-theoretic/probabilistic notions.

[25]See, e.g., Geroch (1970), Geroch (1971), Hawking (1971), and Lerner (1973). For a great discussion, see Fletcher (2016) and Fletcher (2018a).

[26]See, e.g., Hawking (1971, fig. 1 and 2) for illustration and Fletcher (2018a, sec. II) for formal definition.

5 Mathematical Formalization of Stability: Further Directions

In the preceding Section 4.3, we already saw one particularly fruitful formalization of stability: namely, via topology. In this section, we sketch three further avenues for an investigation of stability using formal methods.

(1) *Modal logic*. In addition to topology, modal logic offers another suggestive formalization of stability by interpreting the box operator $\Box$ as stability.[27] Thus, if X is a class of models and $x \in X$ is a particular model, then $X, x \vDash \varphi$ means that model x has property φ, and $X, x \vDash \Box\varphi$ means that model x has property φ stably, i.e., all relevantly similar models also have property φ. To give an example of this, we can consider the class of models of ZFC and a model x is 'relevantly similar' to a model y if y is a forcing extension of x. Hamkins and Löwe (2008) prove that, once made fully precise, the resulting modal logic—i.e., the modal logic of forcing—is S4.2. Moreover, many further instances of the modal logic interpretation of stability are discussed in [*reference redacted*], and Fitch's lemma is used to derive an impossibility result: that several prima facie plausible principles about stability cannot be jointly satisfied.

(2) *Persistent homology*. Section 4.3 formalized the question of which notion of similarity to choose into the question of which topology to choose on the class X of models of some phenomenon. If there is no canonical choice, we can regard this question as a (meta-) instance of the principle of stability: Each relevant topology τ on X is now one model of the (meta-) phenomenon 'similarity'. We then are concerned with the property

$$\psi(\tau) := \text{property } \varphi \text{ is stable in topology } \tau,$$

where φ is the property under discussion in the original phenomenon. If ψ is itself stable, i.e., holds for a wide range of different topologies τ, we are justified to infer it for the intended notion of similarity, but if it is not stable, we are not justified. This question of which topological features persist when varying the topology is studied in persistent homology—a central method in topological data analysis. (For an overview, see Edelsbrunner and Harer (2008), Otter et al. (2017), and Weinberger (2011).) In particular, one considers the features that are tracked by homology groups (e.g., the number of components or holes). If such a feature persists for a wide range of topologies, it is considered significant, and otherwise it is regarded as noise.[28]

(3) *Stone duality*. Another mathematical field that is well poised to better understand the space of models of a theory is Stone duality. It describes a

[27] There are well-known connections between modal logic and topology: on the topological semantics for modal logic, the clause for 'x having property P stably' from Section 4.3 is precisely the clause for $x \vDash \Box P$. See van Benthem and Bezhanishvili (2007) for details.

[28] Technically, one assumes that the considered topological spaces form a nested sequence indexed by the real numbers, and a feature persists for a long time if it persists for a large interval of indices.

precise duality between the 'syntactic world' of theories on the one hand and the 'semantic world' of spaces of models on the other hand. In other words, every theory uniquely determines a topology on the space of its models, and every space of models uniquely determines its theory. Let us illustrate this idea in the propositional case. (For generalizations to first-order logic, see Makkai (1987) and Awodey and Forssell (2013).) Given a theory T, we regard it as a Boolean algebra: namely, the set of all sentences modulo being provably equivalent under T. A model of T is then an ultrafilter of this Boolean algebra, i.e., a maximally T-consistent set of sentences. Now we can put a natural topology on the set X of all models of T as follows. Each sentence φ determines a natural similarity degree: given a model x, consider a model y similar to x to that degree if y agrees with x on the truth-value of φ. This generates a topology on X, which turns it into a so-called Stone space.[29] Conversely, every collection of models with a topology that renders it a Stone space also determines a Boolean algebra: namely the Boolean algebra of closed-and-open sets (which, intuitively, are the 'basic' similarity degrees just described). For references on Stone duality, see Johnstone (1982) and Gehrke and van Gool (2024); for the conceptual development, also see Abramsky (1991), Fletcher and Lackey (2022), and Lawvere (1969); and for applications of this ballpark of ideas to the debate on 'models vs theory', see Hudetz (2017, 2019).

6 Conclusion

We conclude with a brief summary. We started with the essay question of whether there is an epistemological principle that renders coherent Gödel's three views on mathematics, time, and theism. Our answer was: the principle of stability. For each view, we provided a scientific modeling context in which the principle of stability entails the view held by Gödel. Specifically, for mathematics, absoluteness results in ZFC entail the stable existence of certain mathematical objects; for time, Gödel's universe shows the non-stable existence of global time; and for theism, Gödel's ontological proof shows the stable existence of an entity that has all positive properties. We then looked at the dynamics of context change. Changing the theory is suggested by Gödel's programmatic search for new axioms of set theory, which might justify more objects; changing to other properties revealed a modal argument pattern in general relativity; and changing the notion of similarity showed that Gödel's argument idealism about time can be formally interpreted as being sensitive to the choice of topology on the class of spacetimes. Finally, we identified three promising directions for further investigations of the stability using formal methods.

[29] Hence, properties expressible in the language of the theory are automatically stable. However, subsets $P \subseteq X$ that are not open (e.g., closed and not open) represent properties that are not stable.

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