

# Unifying and Validating Some Ideas of Kurt Gödel

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## Abstract

In this paper, I show how an axiomatic metaphysics, *object theory* (OT), offers a formal framework for understanding and unifying some of the views expressed in Gödel's discussions concerning (a) the nature of mathematics and metaphysics, (b) the works and ideas of Edmund Husserl and how they relate to mathematics, (c) an ontological argument for the existence of God, and (d) an argument for the nonexistence of time.

## 1 Introduction

In his writings, both published and unpublished, and in his conversations with Hao Wang, Kurt Gödel discussed (a) the nature of mathematics and metaphysics, (b) the works and ideas of Edmund Husserl and how they relate to mathematics, (c) an ontological argument for the existence of God, and (d) an argument for the nonexistence of time. In this paper, I show how *object theory* (OT) offers a formal framework for understanding and unifying some of the views expressed in these discussions.

OT has been developed in a number of publications since the 1980s. In what follows, I shall introduce the features of OT that are needed to understand the Gödel's discussions as the occasion arises. But it is to be noted, at the outset, that OT is an axiomatic metaphysics that includes no mathematical notions as primitive and no mathematical axioms. Instead, OT systematizes a domain of abstract objects and abstract relations from which an analysis of the language of mathematics can be given. Three recent publications about that analysis, and about the philosophy of mathematics OT gives rise to, will be particularly relevant in what follows. In Leitgeb, Nodelman, & Zalta 2025, it is shown how OT

can be extended with analytic truths (expressing that the theorems of an arbitrary mathematical theory  $T$  are true in  $T$ ) so as to:

- identify the abstract objects denoted by the individual constants of  $T$ , and identify the abstract relations denoted by the predicates of  $T$ , and
- assign truth conditions to the theorems of  $T$  in terms of the abstract entities that serve as the denotations of the terms/predicates of  $T$ .

In Nodelman & Zalta 2024, there is a derivation of second-order Peano Arithmetic and the existence of an infinite cardinal ( $\aleph_0$ ) from OT without the addition of any analytic truths or mathematical axioms. And in Zalta 2024, one can find a metaphilosophical approach to unifying the many different philosophies of mathematics, such as Platonism, Structuralism, Inferentialism, Logicism, etc., as different interpretations of OT's formalism.

These papers, and others, will serve as the documentation for the analysis of Gödel's discussions of mathematics and metaphysics developed in what follows. Though there are aspects of Gödel's view that won't be preserved, enough of his views will be validated so as to make a strong *prima facie* case that OT is the kind of theory that undergirds Gödel comments on mathematics and metaphysics. Once we have a thorough understanding of this (Sections 2 and 3), we then discuss how OT helps to explain why Gödel took an interest in Husserl (Section 4), his ontological argument for the existence of God (Section 5), and his argument against the existence of time (Section 6).

## 2 Gödel's View About Axiomatic Metaphysics

Reporting on the tenor of their conversations in general, Wang observed (1996, 244):

In discussions with me, Gödel stressed the central importance of the axiomatic method for philosophy. He did not elaborate his conception of the method, except that he often gave the impression that the task is to find the primitive concepts and then try to see the true axioms for them directly by our intuition.

And Gödel indicated (\*1960/? [Wang 1996, 316]):

13. There is a scientific (exact) philosophy and theology, which deals with concepts of the highest abstractness; and this is also most highly fruitful for science.

Moreover, Gödel had an idea of where such a metaphysics would start. He told Wang (1996, 168):

5.3.17 The basis of everything is meaningful predication, such as  $Px$ ,  $x$  belongs to  $A$ ,  $xRy$ , and so on. Husserl had this. Hegel did not have this; that is why his philosophy lacks clarity. ...

These three remarks are faithfully captured by OT. It is an axiomatic system consisting of the principles that systematize and govern abstract objects and abstract relations generally. It is based on *two* fundamental notions of predication by extending second-order quantified modal logic with a second form of predication. In addition to the classical form of predication in the predicate calculus, namely  $F^n x_1 \dots x_n$  (read: objects  $x_1, \dots, x_n$  exemplify relation  $F^n$ ), OT additionally employs atomic formulas of the form  $x F^1$  (read: object  $x$  encodes property  $F^1$ ).

This idea of an object  $x$  encoding a property  $F$  derives from Mally 1912, who suggested that abstract objects are distinguished and defined by the properties that *determine* them (*sein determinieren*), not by the properties that they exemplify (*erfüllen*). (See the Appendix for some extended quotations from Mally 1912.) Henceforth, we say  $x$  ‘encodes’  $F$  where Mally would say  $x$  ‘is determined by’  $F$ . Intuitively, for one to think about an abstract object, one must know in principle what its defining properties are. For example, according to Mally, The (Euclidean) Triangle (in the abstract) is an object that encodes all and only the properties necessarily implied by *being a triangle*. So while The Triangle encodes properties such as *being a plane figure*, *having three sides*, *having three interior angles*, etc., it won’t encode *being equilateral*, or *being isosceles*, etc. Thus, it encodes an ‘incomplete’ number of properties, though by the laws of logic, it *exemplifies*, for every property  $F$ , either  $F$  or the negation of  $F$ . So, since The Triangle is abstract, it will exemplify *not having a shape*, *not having a color*, *being thought about by the reader now*, etc.

If we use the predicate ‘ $A!$ ’ (‘*being abstract*’) to help us assert that  $x$  exemplifies being abstract (‘ $A!x$ ’), then Mally’s idea is captured formally in OT by the following comprehension principle, which asserts that for any (expressible) group of properties (i.e., for any definable condition on

properties), there exists an abstract object that encodes just those properties and no others:

$$\exists x(A!x \ \& \ \forall F(xF \equiv \varphi)), \text{ provided } x \text{ isn't free in } \varphi \quad (1)$$

This principle is supplemented by the principle that abstract objects are identical whenever they (necessarily) encode the same properties ( $A!x \ \& \ A!y \rightarrow (x = y \equiv \Box \forall F(xF \equiv yF))$ ). From these principles it follows that:

$$\exists! x(A!x \ \& \ \forall F(xF \equiv \varphi)), \text{ provided } x \text{ isn't free in } \varphi \quad (2)$$

where  $\exists! x \psi$  asserts that there exists a unique  $x$  such that  $\psi$ . These principles are supplemented by (a) an axiom asserting that the properties encoded by an object are necessarily encoded ( $xF \rightarrow \Box xF$ ), and (b) a theory of hyperintensional properties, relations, and propositions that is formulable in second-order logic, in terms of (i) the second-order comprehension principle for relations, restricted so that new relations can’t be built out of encoding conditions, and (ii) a definition that stipulates identity conditions for relations. In the simplest case of unary relations (i.e., properties), the latter asserts that properties are identical just in case they are necessarily encoded by the same objects, i.e.,  $F = G \equiv_{df} \Box \forall x(xF \equiv xG)$ . In OT, the definiens doesn’t entail that  $\Box \forall x(Fx \equiv Gx)$ , and so the resulting theory of properties is hyperintensional.

Now to see how OT, as formulated, might further satisfy the desiderata that Gödel proposed for a precise metaphysics, note that Wang describes/quotes Gödel’s position as follows (1974, 85; 1996, 167):<sup>1</sup>

Philosophy as an exact science should do for metaphysics as much as Newton did for physics.

Then, Wang (1996, 167) records the following remark:

5.3.11 The beginning of physics was Newton’s work of 1687, which needs only very simple primitives: force, mass, law. I look for a similar theory for philosophy or metaphysics. Metaphysicians believe it possible to find out what the objective reality is; ...

And in Wang 1996 (308):

<sup>1</sup>In the 1974 work, Wang describes Gödel’s position this way, but in the 1996 work, Wang puts the remark into quotation and substitutes ‘for’ for the two occurrences of ‘to’. Cf. Wang 1974 (85) and Wang 1996 (167).

9.3.20 Philosophy is more general than science. Already the theory of concepts is more general than mathematics. ...

Clearly, these passages suggest that Gödel believed that some part of philosophy could be formulated as a rigorous discipline. And the various applications of OT demonstrate the extent to which OT is a metaphysical theory that forms the basis of an exact science. This claim can be best documented by a brief perusal of the unpublished, online monograph *Principia Logico-Metaphysica* (Zalta m.s.), which compiles, in one place, all of the formal consequences of OT developed in various publications over the years.<sup>2</sup> This single source makes it clear that for each application (e.g., to situations, possible worlds, natural classes, natural numbers, mathematical theories, etc.), many of the theorems of OT are derivable philosophical principles that other philosophers stipulate.

It is not being suggested that Gödel, in the above quoted remarks, had OT in mind. But rather that OT does satisfy his desiderata for a ‘scientific (exact) philosophy’. If predication is as fundamental as Gödel says, and an axiomatization of predication systematizes a widely-applicable domain of abstract objects and abstract relations, then OT goes some way towards satisfying the desiderata Gödel has offered. Indeed, if we additionally think of abstract objects as *concepts*, then OT also offers a way of systematizing Gödel’s talk of abstract concepts and their connection to mathematics.

### 3 Realism About Mathematics: OT vs. Gödel

Some further evidence, and some counterevidence, for the foregoing claims can be found when we consider the Platonist interpretation of OT and its application to mathematical language. As mentioned earlier, OT, as a formalism applied to the analysis of mathematical language, can be interpreted in a number of ways (Zalta 2024, §4). Under the Platonist interpretation, each instance of principles (1) (and (2) asserts the *existence* of a (unique) abstract object. By the laws of definite descriptions, (2) implies that each such instance yields a well-defined, *canonical*

<sup>2</sup>On need only look at the Table of Contents for Chs. 7–15, and the List of Important Theorems, pp. xviii–xxiii, many of which have been computationally verified in Kirchner 2022. The proofs are in Isabelle/HOL and Kirchner has put the proofs online in a GitHub repository: <https://aot.ekpyron.org/AOT/AOT/index.html>.

definite description of the form  $\iota x(A!x \& \forall F(xF \equiv \varphi))$ . Then, for each mathematical theory  $T$ , we may deploy descriptions having this form to identify the well-defined mathematical objects and relations of  $T$ . This is spelled out in detail in Leitgeb, Nodelman, & Zalta 2025. The analysis there can be summarized as follows.

From the standpoint of OT, the basic data of mathematics are truths of the form ‘In theory  $T$ ,  $p$ ’ or ‘ $p$  is true in  $T$ ’. OT then analyzes mathematical theories as abstract objects that encode propositions, by encoding properties of the form *being such that*  $p$ , where these latter are represented as  $[\lambda x p]$ .<sup>3</sup> Then OT defines ‘ $p$  is true in  $T$ ’ ( $T \models p$ ) just in case  $T$  encodes  $[\lambda x p]$ , i.e.,

$$T \models p \equiv_{df} T[\lambda x p]$$

Now, for any given theory  $T$ , the theorems of  $T$  can be imported into the language of OT as *analytic truths* about what is true in  $T$ , as follows: if  $T \vdash \varphi$ , then add to OT the analytic truth  $T \models \varphi^*$ , where  $\varphi^*$  is the result of indexing the terms and predicates of  $T$  to  $T$ . Now suppose that  $\kappa$  is a well-defined individual term of theory  $T$ . Then we can identify the mathematical object  $\kappa$  of theory  $T$  as follows:

$$\kappa_T = \iota x(A!x \& \forall F(xF \equiv T \models F\kappa_T)) \quad (3)$$

This asserts that the object  $\kappa$  of theory  $T$  is the abstract object that encodes just the properties  $F$  such that the proposition that  $F\kappa_T$  is true in theory  $T$ . In other words, the object  $\kappa$  of theory  $T$  encodes exactly the properties that  $\kappa$  exemplifies in  $T$ . So, for example, the null set  $\emptyset$  of ZF may now be identified as the abstract object that encodes exactly the properties  $F$  that  $\emptyset$  exemplifies in ZF:

$$\emptyset_{ZF} = \iota x(A!x \& \forall F(xF \equiv ZF \models F\emptyset_{ZF})) \quad (4)$$

<sup>3</sup>These properties are perfectly well-behaved from a logical point of view. They are axiomatized by principles of the  $\lambda$ -calculus, interpreted relationally. So, for example, where  $p$  is the proposition *that Trump is president* ( $Pt$ ), the expression  $[\lambda x Pt]$  denotes the property: being (an  $x$ ) such that Trump is president. Thus, we have the following instance of  $\lambda$ -Conversion (i.e.,  $\beta$ -reduction):  $[\lambda x Pt]y \equiv Pt$ ; that is,  $y$  exemplifies being such that Trump is president if and only if Trump is president. The  $x$  bound by the  $\lambda$  in  $[\lambda x Pt]$  is vacuously bound, and so something exemplifies the property denoted just in case the proposition from which the property is built is true. In general, the 0-ary instance of  $\lambda$ -Conversion yields:  $[\lambda x p]y \equiv p$ , for any  $p$  and  $y$ .

This analysis can be generalized so as to identify the properties and relations of  $T$  as higher-order abstract properties and relations. The type-theoretic version of OT is formulated in terms of a relational type theory, where  $i$  is the type for individuals and  $\langle t_1, \dots, t_n \rangle$  is the type for relations whose arguments have types  $t_1, \dots, t_n$  ( $n \geq 0$ ). Then we can type the terms of the language, type the atomic formulas of OT, and thereby formulate a typed version of (1):

$$\exists x^t (A!^{(t)}x \ \& \ \forall F^{(t)}(xF \equiv \varphi)), \text{ where } x^t \text{ isn't free in } \varphi \quad (5)$$

This asserts, for any type  $t$ , the existence of an abstract object of type  $t$  that encodes just the properties having type  $\langle t \rangle$  that satisfy a condition  $\varphi$  placing a condition on such properties. So if  $t \neq i$ , then  $t$  is a relational type and thus (5) yields, at every such type, abstract relations of that type. Once the typed version of (2) is derived, we may assert the type-theoretic version of (3). That is, the identification principle may be generalized so as to apply to higher-order mathematical relations as well as to mathematical objects:

$$\kappa_T^t = \iota x^t (A!^{(t)}x \ \& \ \forall F^{(t)}(xF \equiv T \models F\kappa)) \quad (6)$$

To see a higher-order instance of this principle, let's identify the membership relation  $\in$  of ZF. This is a relation of type  $\langle i, i \rangle$ , since it is a binary relation among sets, construed as individuals. Then where ' $\in$ ' and the variable ' $x$ ' are of type  $\langle i, i \rangle$ , and ' $A!$ ' and the variable ' $F$ ' are of type  $\langle \langle i, i \rangle \rangle$  (i.e., they denote, or range over, properties of binary relations among individuals), we have the following instance of our identification principle (6):

$$\in_{ZF} = \iota x (A!x \ \& \ \forall F (xF \equiv ZF \models F \in_{ZF})) \quad (7)$$

This asserts that the membership relation  $\in$  of ZF is the abstract relation that encodes exactly the properties of relations which  $\in$  exemplifies in ZF.<sup>4</sup> It is important to remember here that (7) is not a *definition* of the

<sup>4</sup>For example, we know  $ZF \vdash \emptyset \in \{\emptyset\}$ , and so by  $\beta$ -Conversion,  $ZF \vdash [\lambda F \emptyset F\{\emptyset\}] \in$ , where  $F$  here is a variable of type  $\langle i, i \rangle$ . The latter asserts that it is a theorem of ZF that the membership relation exemplifies the higher-order property of relations: being a binary relation on individuals that relates the null set to the unit set of the null set. This becomes imported into OT as  $ZF \models [\lambda F \emptyset F\{\emptyset\}]_{ZF} \in_{ZF}$ . So from the identification of  $\in_{ZF}$  asserted in (7), it follows that  $\in_{ZF}$  encodes the ZF-property  $[\lambda F \emptyset F\{\emptyset\}]_{ZF}$ . In this manner, we can extract from every theorem of ZF, a higher-order property of  $\in_{ZF}$ . OT guarantees that there is an abstract relation that encodes all and only such higher-order properties of relations.

symbol ' $\in$ ', but is a principle that theoretically identifies the membership relation of ZF in terms of the truths of ZF. Again, the details are spelled out in Leitgeb, Nodelman, & Zalta 2025, where the analysis is shown to generalize to arbitrary mathematical theories.

But, although the foregoing is a form of realism about mathematical objects and relations (given that it asserts the existence of abstract objects and relations, and identifies mathematical objects and relations among these abstract entities), one might question whether important elements of Gödel's form of realism are preserved. Before we say what these elements are, note that the present view takes each mathematical theory  $T$  to be (i.e., encode truths) about the domain of objects and relations of  $T$ . It takes the membership relation of ZF to be different from the membership relation of ZF+AC (since they are abstracted from different bodies of truths), and rejects the idea that there is only one correct set theory. The present approach allows us to analyze the meaningfulness of the language used in *arbitrary* mathematical theories, and thus of any consistent theory of sets with a distinctive group of theorems.

Doesn't this appear to be incompatible with Gödel's belief that there is only one correct axiomatization of set theory and that we simply have to keep searching for axioms that will "force themselves upon us as being true" (1947 [1964, 271])? And didn't Gödel conceive of the mind-independence and objectivity of mathematical objects (and in particular, sets) on the model of physical objects, by arguing that we have "something like a perception ... of the objects of set theory" (1947 [1964, 271])? How are we to reconcile the above view with Gödel's claim that the objective existence of the objects of mathematical intuition "is an exact replica of the question of the objective existence of the outer world" (1964, 272)?

To answer these questions, note that Gödel writes (\*1951, 320):

What is wrong, however, is that the meaning of the terms (that is, the concepts they denote) is asserted to be something man-made and consisting merely in semantical conventions. The truth, I believe, is that these concepts form an objective reality of their own, which we cannot create or change, but only perceive and describe.

But surely Gödel would agree that the best way to describe objective reality with a high level of precision and accuracy is in terms of an axiomatic theory of objects and concepts. Since one can adopt a realist (or Platon-

ist) attitude about the comprehension principle for relations in second-order logic, one can also adopt a realist (or Platonist) attitude about the comprehension principle for abstract objects. So axiomatization, then, is one clear way of describing an objective reality.

It should also be noted that Gödel himself did not offer a precise theory of mathematical objects and concepts of the kind OT represents. Gödel's use of the terms 'mathematical concept' and 'abstract concept' were thus pre-theoretical. So if a systematic and widely-applicable metaphysics that satisfies a number of Gödel's desiderata implies that each mathematical theory is about its own domain, then it might be legitimate to adjust Gödel's view a bit 'leftward'. By this, we are recalling Gödel's division of philosophical 'world-views' (*Weltanschauungen*) between the 'rightward' views of spiritualism, idealism, and apriorism, on the one hand, and the 'leftward' views of skepticism, materialism, and positivism, on the other (\*1961/?, 375). Gödel notes (\*1961/?):

But the next step in the development is now this: it turns out that it is impossible to rescue the old rightward aspects of mathematics in such a manner as to be more or less in accord with the spirit of the time.

But one of the themes of \*1961/? is a reconciliation of the leftward and rightward world-views, for he says (381):

As far as the rightness and wrongness, or, respectively, truth and falsity, of these two directions is concerned, the correct attitude appears to me to be that the truth lies in the middle or consists of a combination of the two conceptions.

In what follows, we suggest how the truth 'lies in the middle' of the two conceptions.

Here is one way to move Gödel's underlying conception forward. His view seems to be that a science of abstract objects has to model natural science in every relevant respect. But Linsky & Zalta (1995) argue that we shouldn't model the mind-independence and objectivity of abstract objects by analogy with the mind-independent and objectivity of physical objects. Abstract objects and concepts are not mind-independent and objective in the same way physical objects are.<sup>5</sup> The former, not the latter, can and should be systematized by comprehension principles and

by asserting a theory. The comprehension principle (1) guarantees that abstract entities encode (but don't necessarily exemplify) the properties that satisfy any condition  $\varphi$ . This is, therefore, a *plenitude* principle. So, under a Platonist interpretation, a comprehension principle such as (1) *describes* an objective reality and *grounds* the mind-independence and objectivity of abstract objects. Linsky & Zalta (1995) develop the basic metaphysical and epistemological principles which are appropriate to this kind of mind-independence and objectivity.

If Gödel had encountered a powerful, axiomatic metaphysics which could explain the meaningfulness of mathematical language and which offered a subject matter for arbitrary mathematical theories, he might have taken that view seriously. This is suggested by the following remark (Gödel 1972, 271–2):

By abstract concepts, in this context, are meant concepts which are essentially of the second or higher level, i.e., which do not have as their content properties or relations of concrete objects (such as combinations of symbols), but rather of thought structures or thought contents (e.g., proofs, meaningful propositions, and so on), where in the proofs of propositions about these mental objects insights are needed which are not derived from a reflection upon the combinatorial (space-time) properties of the symbols representing them, but rather from a reflection upon the meanings involved.

OT, which is developed in terms of Mally's second mode of predication, is a potential place to look for what Gödel calls the 'meanings involved', except that these meanings are determined *proof-theoretically*, since they are abstracted from the theorems of each distinct theory. When we examine Husserl's ideas in the next section, we'll also consider how intuition

ject to an appearance/reality distinction, abstract objects are not; physical objects may not have the properties they appear to have, but abstract objects are just the way our theories and conceptions describe them. (b) Abstract objects are not 'out there' in a sparse way waiting to be discovered (in the manner of physical objects), but rather constitute a *plenitude*. And (c), the abstract entities that serve as the denotational content of mathematical terms and predicates are always incomplete; they 'have' (in the sense of 'encode') only their mathematical properties and no others (cf. Dedekind 1888). By contrast, one might regard physical objects, other than the very smallest ones governed by the principles of quantum mechanics, as complete, since they (i) only exemplify their properties, and (ii) are subject to the logical law that for any property  $F$ , they either exemplify  $F$  or exemplify the negation of  $F$ . See Linsky & Zalta 1995 for the argument as to why this conception of abstract objects is the key to naturalizing them.

<sup>5</sup>The reasons Linsky & Zalta give for this are: (a) Whereas physical objects are sub-

of abstract objects might be involved.

So one aspect of Gödel's realism about mathematics is captured by the Platonist interpretation of OT's formalism. On that interpretation, as noted above, each mathematical theory  $T$  is about abstract objects and abstract relations whose encoded properties are precisely those required by the theorems of  $T$ . The Platonist may regard these objects and relations as mind-independent (in the manner of Linsky & Zalta 1995), notwithstanding the fact that they are abstracted from their governing theory, since theories themselves are regarded as abstract objects that encode propositions. The result is that each mathematical term and predicate has a (denotational) content, namely, a well-defined concept (abstract object or abstract relation) that is identifiable in the background ontology of OT. Clearly, then, this is *not* subject to the objection Gödel raised in \*1953/9-III for the early Carnap view, which Gödel describes as follows (\*1953/9-III, 335):

According to this conception (which, in the sequel, I shall call the syntactical viewpoint) mathematics can completely be reduced to (and in fact is nothing but) syntax of language.<sup>[6]</sup> I.e., the validity of mathematical theorems consists solely in their being consequences<sup>[7]</sup> of certain syntactical conventions about the use of symbols,<sup>[8]</sup> not in their describing states of affairs in some realm of things. Or, as Carnap puts it: *Mathematics is a system of auxiliary sentences without content or object.*

OT does assign mathematical terms and predicates a denotational content, and assigns mathematical theorems a determinate meaning. It thereby treats mathematics as having a content and an object.

Moreover, OT validates the following idea (Gödel, \*1959/9-V, 9):

Mathematical propositions ... do not express physical properties of the structures concerned [in physics], but rather properties of the concepts in which we describe those structures.

To see how, recall that the identification of  $\in_{ZF}$  in (7) analyzes this binary relation in terms of the *properties of binary relations* that are attributed to  $\in_{ZF}$  in the theorems of ZF. So we've identified the concept  $\in_{ZF}$  by way of its (encoded) properties.

Note that the above quote continues (*ibid.*):

But this only shows that the properties of those concepts are something quite as objective and independent of our choice as physical properties of matter. This is not surprising, since concepts are composed of primitive ones, which, as well as their properties, we can create as little as the primitive constituents of matter and their properties.

But on the Platonist interpretation of OT's formalism, the work of the mathematician, in axiomatizing mathematical objects and relations, is to carve out, by means of the axioms, which of the possible mathematical objects and relations are under consideration. OT provides the background ontology of possible mathematical objects and relations. The axioms of ZF then fix the meaning of  $\emptyset_{ZF}$  and  $\in_{ZF}$  required by those particular axioms. (7), for example, fixes the identity of  $\in_{ZF}$  among the possible abstract relations. Thus, mathematical objects, as identified, are objective and exist independently of us.

One final consideration is whether there are parts of mathematics that are derivable from the pure laws of metaphysics *without* the addition of any analytic truths. Interestingly, the work in Nodelman & Zalta 2024 validates this idea. In that paper, second-order Peano Arithmetic, recursive function theory, and the existence of  $\aleph_0$  are all derivable from the first principles of OT, supplemented with the axiom that a certain ordering condition, definable without mathematical primitives, constitutes a relation. These results confer a distinguished status on Peano Arithmetic.<sup>6</sup>

In the foregoing, we have attempted to show that there are ways to see OT as validating the ideas associated with Gödel's form of realism and that adjustments can be made to those parts of his realism which seem incompatible with OT. As we noted in quoted passages above, Gödel himself thought that the rightward end of the spectrum of world-views could no longer be consistently maintained, and so some movement towards the left is inevitable. Our argument and approach has been to adjust the position he adopted 'slightly further leftward', in the attempt to further reconcile the two general world-views. With just a few adjustments towards the left-side of the spectrum of world-views, one can pro-

<sup>6</sup>OT also provides a consistent, but 'flat', theory of natural classes, since classes can be defined as the extension of concepts and the principal axioms of set theory, *other than* the power set axiom, can be derived as theorems. See Anderson & Zalta 2004, and Zalta m.s. (Chapter 10, Section 10.6).

vide Gödel with a realistic metaphysics that maximizes the satisfaction of his desiderata, which otherwise may pose an unsolvable constraint satisfaction problem.

## 4 Gödel's Remarks about Husserl

In this section, we explore Gödel's turn to the works of Edmund Husserl in his attempt to better understand the nature of mathematics and how mathematics might involve the "clarification of meaning" by a theory of concepts. A discussion of Husserl's ideas also provides a natural context for discussing mathematical intuition. Though Gödel studied Husserl in some detail, he was unaware, or at least never seems to have remarked upon, the connection between Ernst Mally's work of 1912 (which forms the basis of OT) and Husserl's work of 1913. Though a number of philosophers have studied Gödel's understanding of Husserl (Føllesdal 1995; Tieszen 1992, 1998), none have remarked on the connection with Mally either. The connection is this: Mally's notion of encoding (*sein determinieren*) proves crucial to making Husserl's view, about the directness of our mental states, precise and amenable to formalization. In this section, we'll first look at what Gödel says about Husserl's philosophy and then investigate the connection between Husserl 1913, Mally 1912, and OT.

Gödel's remarks about Husserl were made in various unpublished works and in conversation with Hao Wang. For example, referring to Husserl's work generally but with special attention to Husserl 1913, Gödel says (\*1961/?, 383):<sup>7</sup>

... there exists today the beginnings of a science which claims to possess a systematic method for such clarification of meaning, and that is the phenomenology founded by Husserl. Here clarification of meaning consists in concentrating more intensely on the concepts in question by directing our attention in a certain way, namely, onto our own acts in the use of those concepts, onto our own powers in carrying out those acts, etc. In so doing, one must keep clearly in mind that this phenomenology is not a science in the same sense as

the other sciences. Rather it is [or in any case should be] a procedure or technique that should produce in us a new state of consciousness in which we describe in detail the basic concepts we use in our thought, or grasp other, hitherto, unknown, basic concepts.

Though I'm not sure exactly what Gödel had in mind here, the context suggests that Husserl's views provide insight into some basic questions in the philosophy of mathematics. But Husserl's discussion of phenomenology has a gap that is filled by Mally's distinction between being determined by a property and satisfying/exemplifying a property, i.e., by the introduction of the encoding mode of predication.<sup>8</sup> Here is how.

Let's 'bracket' the external world, as Husserl suggests, and consider the phenomenology of the cognitive state in which we appear to see a tree. We might actually be seeing a tree or mistakenly seeing a tree or dreaming about seeing a tree. Husserl would say, in the cases where we are mistaken or dreaming, that although the world doesn't contain the tree, our mental state has a sense (*noematische Sinn*) that is characterized by the property *being a tree*. But clearly, this notion of *characterization* can't be the exemplification form of predication, since the content of our mental state doesn't exemplify being a tree – the tree does. (See the extended quotations from Husserl in the Appendix, where he says that the tree can burn up, but the *perceived tree as perceived* cannot.) Husserl, though, doesn't have a precise theory of how it is that the property *being a tree* characterizes the content of such mental states. In his writings, Husserl uses inverted commas (quote marks) to signal that the property of being a tree is applicable in a special way. But note that his inverted commas can't change the meaning of the word 'tree', for if the word 'tree' changes its meaning when Husserl both places it in quote marks and uses the result to describe the sense of our mental state, then it would be a mystery how the mental state could direct us towards *trees*.

Mally's notion of being determined by (or encoding) a property provides a formalizable interpretation of Husserl's use of quote marks. Mally agrees with Husserl that nothing exemplifies the property of being a tree in those cases where our experience is mistaken, or part of a dream, etc., and he would agree that we can describe our mental states in terms of an intermediate object. Mally would call that intermediate object an

<sup>7</sup>Føllesdal (1995, 367) notes that Gödel owned all of Husserl's major works, and follows this up with specifics in note b. But he then says (368) that "Generally, Gödel is most appreciative of" of Husserl 1913.

<sup>8</sup>The argument for this was first developed in detail Zalta 1998, and briefly rehearsed in Zalta 2002.

‘abstract determinate’, while Husserl would call it a ‘noematic sense’. But following Mally, we can say that in the cases of experiencing a tree, whether the experience is veridical or mistaken, the content of our mental state can be characterized as *encoding* the property of being a tree.

Thus, Mally’s second mode of predication can do the work of Husserl’s inverted commas. When Husserl says that “‘tree’” (i.e., the word ‘tree’ in inverted commas) characterizes the noematic sense of our perception of a tree, instead of thinking that the inverted commas change the meaning of the word ‘tree’, we can suppose that the inverted commas change the mode of predication. The very same property, *being a tree*, is *exemplified* by the tree in nature (in the case of veridical perception), but is *encoded* by the noematic sense. This explains how, for Husserl, the noematic sense gives our mental state a direction towards things in the world, even in the case where the perception is not veridical. In other words, whenever Husserl correctly uses words in inverted commas to characterize the noematic sense, we may interpret him as asserting that the noematic sense encodes the property expressed by those words. So the logic we’ve developed for encoding a property can serve as a logic for (or theory of) Husserl’s noematic senses.

This discussion reveals how the ideas of Mally might inform Gödel’s understanding of Husserl. For *Husserl’s phenomenological method applies generally to all forms of cognition, both to experience of physical objects and to thoughts about abstract entities such as mathematical objects and relations*. Føllesdal describes Husserl’s understanding of intuition as follows (Føllesdal 1995, 370):

According to him [Husserl], intuition, as well as perception, is of objects. There are two kinds of intuition, according to Husserl: perception, where the object intuited is a physical object, and categorical, or eidetic intuition, where the object is an abstract entity. The object, whether it be concrete or abstract, is always intuited as having various properties and bearing relations to other objects. These properties and relations can be singled out for our attention in acts of judgment, in which the object is judged to have such and such features. Intuition of objects is hence more basic for Husserl than judgments: intuition provides the evidence for judgments. It is not clear whether Gödel shared this view that intuition of objects is more basic.

Since the object, whether concrete or abstract, “is always intuited as having various properties and bearing relations”, these properties and relations have to characterize the contents of the mental state involved in those judgments by way of the encoding mode of predication. For the intuitions involve noematic senses, not the physical objects nor the abstract objects themselves. In the case of eidetic intuitions of abstracta, we have only the abstract objects themselves. With abstract objects, however, knowledge by acquaintance and knowledge by description *collapse*, for as Linsky & Zalta note (1995, 547), “all one has to do to become acquainted *de re* with an abstract object is to understand its descriptive, defining condition”. This is captured by principle (6) and its instances (4) and (7) – all one has to do to become acquainted with  $\emptyset_{ZF}$  or  $\in_{ZF}$  is to understand its identifying description.

But though we identified the *denotations* of mathematical terms in Section 2, there are additional abstract entities that can serve as the *Fregean senses* of these terms and as the constituents of noematic senses – these are abstract entities that encode only the properties by which one cognizes the denotations of the terms. Clearly, though, such abstract entities won’t typically encode every property encoded by the denotation of the term.

So Gödel’s claim (\*1953/9-V, 359):

The similarity between mathematical intuition and a physical sense is very striking.

can be validated to this extent: in the case of physical experiences, the content of our intuitions can be characterized by abstract entities that encode ordinary properties of the kind *exemplified* by the physical objects to which our mental state may be directed; in mathematical intuition, however, the content of our intuitions can be characterized by abstract entities that encode some of the properties of the kind *encoded* by the mathematical objects to which our mental state is directed.

So, adopting Frege’s distinction in meaning between the sense and denotation of a term, we can say that the abstract entities in OT can serve both as the denotations of mathematical terms and predicates and as the senses of mathematical terms and predicates (in both Husserl’s and Frege’s notion of ‘sense’). In the present work, I’ve not offered a precise account of how the intuitions a mathematician might have drive the formation of the axioms for the mathematical theory they are develop-

ing. Theory formation is a topic for another occasion. But at least we have developed some understanding of the nature of intuitions, insofar as how Mally's distinction in predication can link Husserlian noematic senses with Gödel's understanding of intuition.

## 5 Gödel's Ontological Argument

What is driving Gödel's ontological argument and how does OT help us to understand the essential reasoning underlying his argument? The basic problem for any ontological argument is to argue that God's existence is required (in light of additional premises) by the definition of God. For example, in Anselm's case, the definition of God, namely, that than which nothing greater can be conceived, is supposed to imply God's existence (usually via a reductio that appeals to a premise about *greater than* and a premise about a consequence of something *not* existing). In Descartes' case, the definition of God, namely, that which has all perfections, is similarly supposed to imply God's existence (in light of the premise that existence is a perfection). But in *modal* ontological arguments, such as Gödel's, the very possibility that the definition of God is realized is supposed to imply that the definition is in fact realized, i.e., imply God's existence. In Gödel's argument, this can be seen (a) in the definition of what it is for  $x$  to be God-like, namely, that  $x$  exemplify every positive property, and (b) in the axiom that asserts that if a property is positive, then it is necessarily positive.

A summary of the formulation and subsequent investigations into Gödel's argument can be found in Benz Müller & Scott 2025. As noted there, a key feature of the original argument (that Gödel might have welcomed, though most philosophers would not) is that the axioms lead to *general* modal collapse, i.e., to a proof that  $\varphi \rightarrow \Box\varphi$ , for any formula  $\varphi$ . And from this it follows by the Rule of Necessitation (RN) that  $\Box(\varphi \rightarrow \Box\varphi)$ .

But let us abstract away from the specific axioms that Gödel presents and examine the two basic intuitions that seem to be driving modal ontological arguments; these involve a much more limited form of modal collapse, restricted to one proposition. The two intuitions are:

- If God exists, then necessarily God exists, and this conditional fact is itself necessary, since it is definitive of the concept of God.

- God possibly exists.

Clearly, the first of these intuitions already embodies a limited form of modal collapse, namely, a modal collapse restricted to the claim that God exists. And these two facts imply, in S5 modal logic, that God exists.

To see this, note that we can represent the two intuitions, respectively, as follows, where ' $G$ ' denotes the proposition that God exists:

$$\Box(G \rightarrow \Box G) \quad (8)$$

$$\Diamond G \quad (9)$$

Note also that these two basic principles are simplified versions of Theorems Th2 and Th4 in Benz Müller & Scott 2025.<sup>9</sup> The argument that God's existence follows from these two claims appeals to the  $K\Diamond$  principle, which is a modal theorem that asserts  $\Box(\varphi \rightarrow \psi) \rightarrow (\Diamond\varphi \rightarrow \Diamond\psi)$ . Now consider the following instance of  $K\Diamond$ :

$$\Box(G \rightarrow \Box G) \rightarrow (\Diamond G \rightarrow \Diamond\Box G) \quad (10)$$

It then follows from (8) and (10) that  $\Diamond G \rightarrow \Diamond\Box G$ . But, then, from this and (9), it follows that  $\Diamond\Box G$ . But it is a theorem of S5 that  $\Diamond\Box\varphi \rightarrow \varphi$  (this is the  $B\Diamond$  principle). So, given the instance  $\Diamond\Box G \rightarrow G$ , it follows that  $G$ .

This, then, is the root of the derivation of actual existence from possible existence. The point here is not whether the intuitions represented by (8) and (9) are true,<sup>10</sup> but rather that some proposition has to be modally collapsed in order to move from God's possible existence to God's (actual) existence.

This is where OT provides an analogous form of reasoning without any of the untoward consequences. As we noted in Section 2, it is an axiom of OT that if an abstract object  $x$  encodes a property, then it does so necessarily, i.e., that  $xF \rightarrow \Box xF$ . Given this axiom, it follows by Rule RN that  $\Box(xF \rightarrow \Box xF)$ , and in S5, it further follows that  $\Box(\Diamond xF \rightarrow xF)$  is a theorem. That is, necessarily, if an abstract object possibly encodes a property, it in fact encodes that property.

<sup>9</sup>In Benz Müller & Scott 2025, Th2 is  $Gx \rightarrow \Box\exists^E yGy$ , which by Rule RN, implies  $\Box(Gx \rightarrow \Box\exists^E yGy)$ . This corresponds to (8). Th3 is:  $\Diamond\exists^E xGx$ . This corresponds to (9).

<sup>10</sup>Indeed, it seems unlikely that they are conclusive. For if one is not a supertheist and accepts that it is possible God *doesn't* exist, then when we replace (9) with the claim that possibly God doesn't exist ( $\Diamond\neg G$ ), then one can just as easily conclude that God doesn't exist ( $\neg G$ ). To see this, note that from the premise  $\Diamond\neg G$ , it follows that  $\neg\Box G$ . Independently, it follows from (8) by the T schema, that  $G \rightarrow \Box G$ . Hence by modus tollens,  $\neg G$ .

If we consider the two required ingredients for a modal ontological argument from the standpoint of OT, we can easily see that the premise that something either does or might have every positive property is too strong. OT tells us that, as a matter of comprehension, one can assert the existence of a unique abstract object that *encodes* all and only positive properties. That is, given the notion of a positive property, it is an instance of (2) that:

$$\exists!x(A!x \ \& \ \forall F(xF \equiv \text{Positive}(F)))$$

And, of course, in OT, if something encodes every positive property, it necessarily does so, by the axiom  $xF \rightarrow \Box xF$ . Despite these facts, it does *not* follow that anything either exemplifies or possibly exemplifies every positive property. OT tells us that the move from the conception of God as an abstract object (whereby he encodes the positive properties), to the existence of God as an object that exemplifies those properties, is not valid.<sup>11</sup>

But these last facts establish how OT forges a connection between the key pieces of reasoning in Gödel's argument for the existence of God (namely, modal collapse and the necessary existence of a God-like object) and mathematical objects. For (a) it is part of the nature of mathematical objects that their mathematical properties are modally collapsed, and (b) under the Platonist interpretation, mathematical objects necessarily exist. Since every mathematical object is identified in OT as an abstract object that encodes certain properties, it follows that the properties that each mathematical object encodes are necessarily encoded by that object. And it is easy to see that abstract objects necessarily exist. As we noted earlier, OT's central theorem is (2), i.e., the comprehension principle asserting the existence of unique abstract objects. If we apply RN to (2), then it is a theorem of OT that *necessarily*, there is a (unique) abstract

object that encodes just the properties  $F$  that satisfy condition  $\varphi$ , for any condition  $\varphi$  on properties, i.e.,

$$\Box \exists!x(A!x \ \& \ \forall F(xF \equiv \varphi))$$

So, for each instance of comprehension used to identify a mathematical object, the unique abstract object asserted to exist necessarily exists.

On the Platonistic interpretation of OT's formalism, we can conclude that mathematical objects are necessary beings that are independent of us and that whatever mathematical properties they encode at any possible world are properties they encode at every possible world. So, without digging into the specific axioms that Gödel's deploys in the ontological argument, the basic modal reasoning from possibility to actuality is preserved with respect to the defining properties of mathematical objects in OT.

## 6 Gödel's Argument Against the Existence of Time

One further fact about modally collapsed propositions puts us into a position to better understand Gödel's argument against the existence of time. And that is, from  $\Box(\varphi \rightarrow \Box\varphi)$ , it follows that  $\Box(\neg\varphi \rightarrow \Box\neg\varphi)$ . In other words, if a proposition is modally collapsed, so is its negation.<sup>12</sup> And from the fact that  $\Box(\psi \rightarrow \Box\psi)$  is equivalent to  $\Box(\Diamond\psi \rightarrow \psi)$  in S5, it follows that  $\Box(\Diamond\neg\varphi \rightarrow \neg\varphi)$ . That is, necessarily, if the negation of a modally collapsed proposition is possible, then the negation is simply true.

With these results, we can explore a crucial step in Gödel's argument against the existence of time. In 1949a (206–7),<sup>13</sup> Gödel argues for the

<sup>11</sup>In the penultimate paragraph of the Conclusion to Benzmüller & Scott 2025, we find:

... it is not surprising that the notion of God that he tries to capture with the axioms and definitions presented in his 1970 manuscript on the ontological proof is a maximally abstract and maximally consistent, respectively rational, entity, ...

So, if Gödel was indeed attempting to articulate an abstract conception of God, then our moral in the text, namely, that the existence of an object that exemplifies every positive property can't be inferred from the existence of an abstract object that encodes every positive properties, is apropos.

<sup>12</sup>Here is the argument. Assume, as our global assumption, that  $\Box(\varphi \rightarrow \Box\varphi)$ . Now we want to show, without appeal to any contingencies, that  $\neg\varphi \rightarrow \Box\neg\varphi$ , for then our conclusion,  $\Box(\neg\varphi \rightarrow \Box\neg\varphi)$ , follows by Rule RN. So, for conditional proof, assume  $\neg\varphi$ . Now, for reductio, assume  $\neg\Box\neg\varphi$ , i.e.,  $\Diamond\varphi$ . From this and our global assumption, it follows that  $\Diamond\Box\varphi$ , by the K $\Diamond$  principle. But, as we saw previously, the B $\Diamond$  principle, namely  $\Diamond\Box\varphi \rightarrow \varphi$ , is a theorem of S5. Hence  $\varphi$ . Contradiction.

<sup>13</sup>According to a remark of David Malament (Feferman et al. 1986–2003, Volume III, p. 203), this published paper was apparently based on drafts of a longer manuscript, collocated as \*1946/9. A related argument is given in \*1949b, 286–7.

nonexistence of time, i.e., against the objective lapse of time and the failure of change to be objective.<sup>14</sup> He notes that, in the possible worlds where his solutions to Einstein's field equations hold, time travel is possible, though absurdities would arise if a person were to arrive at a past moment and do something to their earlier self which for which they have no memory (or do something worse). His first conclusion is (1949a, 205):

[t]he decisive point is this: that for *every* possible definition of a world time one could travel into regions of the universe which are past according to the definition.<sup>[12]</sup> This again shows that to assume an objective lapse of time would lose every justification in those worlds.

He then begins his argument in earnest (1949a, 206):

Of what use is it if such conditions prevail in certain *possible* worlds? Does that mean anything for the question interesting us whether in *our* world there exists an objective lapse of time? I think it does. For (1) ... there exist ... *expanding* rotating solutions. In such universes, an absolute time also might fail to exist,<sup>[13]</sup> and it is not impossible that our world is a universe of this kind. (2) The mere compatibility with the laws of nature<sup>[14]</sup> of worlds in which there is no distinguished absolute time, and [in which], therefore, no objective lapse of time can exist, throws some light on the meaning of time in those worlds in which an absolute time *can* be defined. For, if someone asserts that this absolute time is lapsing, he accepts as a consequence that whether or not an objective lapse of time exists (i.e., whether time in the ordinary sense of the word exists) depends on the particular way in which matter and its motion are arranged in the world. This is not a straightforward contradiction; nevertheless a philosophical view leading to such consequences can hardly be considered satisfactory.

Here, Gödel has extrapolated facts about the actual world from facts about other possible worlds. The argument seems to be that the existence of an objective lapse of time cannot depend on a contingency (i.e., on the "particular way in which matter and its motion are arranged"). That is:

<sup>14</sup>For a nice summary of the argument, see Yourgrau 2005, 152–154.

Necessarily, if an objective lapse of time exists, it exists necessarily. (11)

So, the claim *an objective lapse of time exists* is modally collapsed, given (11). But, as we saw at the outset of this section, if a proposition is modally collapsed, (a) so is its negation, and (b) if the negation of the proposition is possible, then the proposition is simply true. From (b), we can conclude, from (11) that:

Necessarily, if it is possible that an objective lapse of time fails to exist, then an objective lapse of time simply fails to exist. (12)

So here is where Gödel transitions from possibility to actuality; from the possible nonexistence of an objective lapse of time, the actual nonexistence of an objective lapse of time follows, by (12).

This conclusion suggests that Gödel is reasoning with propositions about abstract objects (abstracted from our intuitive idea of time) and not about empirical propositions. For, by definition, empirical propositions are contingently true if true and contingently false if false. So they can't be modally collapsed given that, by definition, modal collapse entails that truth implies necessary truth and that falsehood implies necessary falsehood.

Consequently, Gödel seems to require some theory of abstract objects in order for his argument against time to be valid. OT provides us with the relevant theory of abstract objects, and thus a coherent ontology for framing his arguments. I've not attempted here to reconstruct Gödel's argument against time in OT; this might require the development of theory of times as abstractions, something that would go beyond the scope of the present paper.<sup>15</sup> But even with such a theory, it isn't clear that one can infer, from the existence of abstract objects that *encode* certain modally collapsed properties, that there exist abstract objects that *exemplify* those modally collapsed properties. And that is what is needed for the argument to go through. So I shall leave this as an open question for future research. But if we couple our conclusions with the idea that OT is also a systematic theory of concepts (Zalta 2000), then we have further evidence that it is a formalism that can usefully unify disparate elements of Gödel's thought.

<sup>15</sup>Though see Zalta 1987, and Chapter 12, Section 12.6 of Zalta m.s.

## Appendix

### From the Work of Mally

In §33 (“Zur Theorie des Begriffes” = “On The Theory of Concepts”), Mally writes (1912, 63):

... Im Gedanken “geschlossene ebene Kurve, deren Punkte von *einem* Punkte gleichen Abstand haben” ist etwas gemeint, das die angenommenen Objektive erfüllt, irgendein Individuum oder Ding aus der Klasse der Kreise ... Was aber im Begriffe unmittelbar gedacht ist, das ist der Gegenstand “geschlossene ebene Kurve, u.s.w.” Dieses begriffliche Abstraktum ist im Begriffe bloß gedacht, nicht auch gemeint. Von ihm ist die Erfüllung der konstitutiven Objektive nicht vorausgesetzt, ... “*der Kreis*” (in abstracto) *erfüllt* die im Kreisbegriffe angenommenen Objektive *nicht*, ... er ist nicht ein Kreis; er fällt deshalb auch nicht unter den Umfang des Kreisbegriffes, gehört der Klasse der Kreise nicht an, ...

Süßbauer and Zalta translate this as follows (Zalta 1998, 11):

... In the thought “closed plane curve, every point of which lies equidistant from *a single point*,” something is meant which satisfies these hypothesized objectives, some individual or thing from the class of circles ... But what is directly conceived in this concept is the object “closed plane curve, etc.” This conceptual abstractum is only conceived in this concept but not meant. That it satisfies the constitutive objectives is not presupposed ... “*the circle*” (in abstraction) *does not satisfy* the hypothesized objectives in the circle-concept, ... it is not a circle; therefore it isn’t in the extension of the circle-concept, it doesn’t belong to the class of circles...

In §33, Mally continues (1912, 64):

Nun ist aber “der Kreis” in abstracto doch ein anderer Gegenstand als etwa “das Dreieck” in abstracto. Was die beiden voneinander unterscheidet, sind die Objektive, die wir als ihre konstitutiven oder definierenden Bestimmungen bezeichnen. Also müssen diese Bestimmungen den Begriffsgegenständen doch in irgendeiner Weise zukommen. Wir sagen: der (abstrakte) Gegenstand “Kreis” ist definiert oder determiniert durch die Objektive “eine geschlossene Linie zu

sein”, “in der Ebene zu liegen”, und “nur Punkte zu enthalten, die von *einem* Punkte gleichen Abstand haben”; er ist als *Determinat* dieser Objektive zu bezeichnen, aber nicht als “implizites” (vgl. §30), da er ja die Objektive nicht erfüllt, sondern, wie man vielleicht sagen könnte, als bloß explizites oder als “Formdeterminat” dieser Objektive.

Süßbauer & Zalta translate this as follows (Zalta 1998, 12):

“The circle” in abstraction is a different object, as for example, from “the triangle” in abstraction. What distinguishes one from the other are the objectives which we call their constitutive or defining determinations. Therefore, these determinations have to belong to the concept-object in some sense. We say: the (abstract) object “circle” is defined or determined by the objectives “to be a closed line”, “to lie in a plane”, and “to contain only points which are equidistant from *a single point*”; we call it the *determinate* of these objectives, but not as an “implicit” one, because it does not satisfy the objectives, but, as one might say, only as an explicit one or as a “formdeterminate” of these objectives.

In §39 (“Abgeleitete Mannigfaltigkeit. Tatsächliche Vollständigkeit bei formaler Unvollständigkeit eines Gegenstandes” = “Derived Variety. Factual Completeness and Formal Incompleteness of an Object”), Mally writes (1912, 76):

Es [“das Quadrat”] erfüllt ja nicht das Objektiv, vier gleiche Seiten zu haben, sondern es ist bloß Formdeterminat dieses Objektivs, und das, was “*das Quadrat*” (in abstracto) tatsächlich erfüllt, ist nichts anderes als eben das Objektiv, Formdeterminat des Quadratseins zu sein, und alles, was darin, daß der Gegenstand eben dieses Formdeterminat ist, impliziert ist. Dazu gehört zum Beispiel, daß dieser Gegenstand in der Tat nicht ein Quadrat ist, daß er überhaupt kein konkreter Gegenstand ist, also insbesondere, daß er keine tatsächliche Ausdehnung, keinen Ort, keine Gestalt, keine Winkel und Seiten besitzt u.s.w.

In the Süßbauer & Zalta translation (Zalta 1998, 15):

It [“the square”] does not satisfy the objective “to have four equal sides”, but it is only a formdeterminate of this objective, and that

which “*the square*” (in abstraction) actually satisfies is nothing other than just the objective “to be the formdeterminate of being a square”, and everything which is implied by the fact that the object is this formdeterminate. This includes, for example, that this object actually is not a square, that it is not a concrete object at all, and especially that it has no actual extension, no spatial location, no shape, no angles or sides, etc.

## From the Work of Husserl

In Husserl 1913 (184), we find:

“In” der reduzierten Wahrnehmung (im phänomenologisch reinen Erlebnis) finden wir, als zu ihrem Wesen unaufhebbar gehörig, das Wahrgenommene als solches, auszudrücken als “materielles Ding”, “Pflanze”, “Baum”, “blühend” usw. Die *Anführungszeichen* sind offenbar bedeutsam, sie drücken jene Vorzeichenänderung, die entsprechende radikale Bedeutungsmodifikation der Worte aus. Der Baum *schlechthin*, das Ding in der Natur, ist nichts weniger als dieses *Baumwahrgenommene als solches*, das als Wahrnehmungssinn zur Wahrnehmung und unabtrennbar gehört. Der Baum schlechthin kann abbrennen, sich in seine chemischen Elemente auflösen usw. Der Sinn aber—Sinn *dieser* Wahrnehmung, ein notwendig zu ihrem Wesen Gehöriges—kann nicht abbrennen, er hat keine chemischen Elemente, keine Kräfte, keine realen Eigenschaften.

Here is the English translation by F. Kersten (1982, 216):

“In” the reduced perception (in the phenomenologically pure mental process), we find, as indefeasibly belonging to its essence, the perceived as perceived, to be expressed as “material thing,” “plant,” “tree,” “blossoming,” and so forth. Obviously, the *inverted commas* are significant in that they express that change in sign, the corresponding radical signification modification of the words. The *tree simpliciter*, the physical thing belonging to Nature, is nothing less than this *perceived tree as perceived* which, as perceptual sense, inseparably belongs to the perception. The tree simpliciter can burn up, be resolved into its chemical elements, etc. But the sense—the sense of *this* perception, something belonging necessarily to its

essence—cannot burn up; it has no chemical elements, no forces, no real properties.

In Husserl 1913 (270), §130 is titled “Umgrenzung des Wesens ‘noematischer Sinn’ ” (“Delimitation of the Essence ‘Noematic Sense’ ”), and we find:

Offenbar ist hiermit ein ganz *fester Gehalt in jedem Noema* abgegrenzt. Jedes Bewußtsein hat sein *Was* und jedes vermeint “sein” Gegenständliches; es ist evident, daß wir bei jedem Bewußtsein eine solche noematische Beschreibung desselben, “genau so, wie es vermeintes ist”, prinzipiell gesprochen, müssen vollziehen können; wir gewinnen durch Explikation und begriffliche Fassung einen geschlossenen Inbegriff von formalen oder materialen, sachhaltig bestimmten oder auch “unbestimmten” (“*leer*” vermeinten<sup>16</sup>) “*Prädikaten*”, und diese in ihrer *modifizierten Bedeutung* bestimmen den “*Inhalt*” des in Rede stehenden Gegenstandskernes des Noema.

In Kersten’s translation (1982, 312–13):

With this, obviously, a quite *fixed content in each noema* is delimited. Each consciousness has its *What* and means “its” objective something; it is evident that, in the case of each consciousness, we must, essentially speaking, be able to make such a noematic description [of “its” objective something] “precisely as it is meant”; we acquire by explication and conceptual comprehension a closed set of formal or material, materially determined or “undetermined” (“*emptily meant*”<sup>17</sup>) “*predicates*” and these in their *modified signification* determine that [the] “*content*” of the object-core of the noema which is spoken of.

Finally, in Husserl 1913 (270–71), §131 is entitled “Der ‘Gegenstand’, das ‘bestimmbare X im noematischen Sinn’ ” (“The ‘Object’. The ‘Determinable X in the Noematic Sense’ ”):

<sup>16</sup>There is a footnote to Husserl’s text here which reads:

Diese Leere der Unbestimmtheit darf nicht mit der Anschauungsleere, der dunkeln Vorstellung vermengt werden.

<sup>17</sup>Kersten’s translation of the footnote that occurs in the text at this point reads:

This emptiness of undeterminedness should not be confused with being devoid of intuition, the emptiness of the obscure objectivation. (p. 313)

Die Prädikate sind aber Prädikate von “*etwas*”, und dieses “*etwas*” gehört auch mit, und offenbar unabtrennbar, zu dem fraglichen Kern: es ist der zentrale Einheitspunkt, von dem wir oben gesprochen haben. Es ist der Verknüpfungspunkt oder “Träger” der Prädikate, aber keineswegs Einheit derselben in dem Sinne, in dem irgendein Komplex, irgendwelche Verbindung der Prädikate Einheit zu nennen wäre. Es ist von Ihnen notwendig zu unterscheiden, obschon nicht neben sie zu stellen und von ihnen zu trennen, so wie umgekehrt sie selbst *seine* Prädikate sind: ohne ihn undenkbar und doch von ihm unterscheidbar. (pp.

In Kersten’s translation (1982, 313):

The predicates are, however, predicates of “*something*,” and this “*something*” also belongs, and obviously inseparably, to the core in question: it is the central point of unity of which we spoke above. It is the central point of connexion or the “bearer” of the predicates, but in no way is it a unity of them in the sense in which any complex, any combination of the predicates would be called a unity. It is necessarily to be distinguished from them, although not to be placed alongside and separated from them; just as, conversely, they are *its* predicates: unthinkable without it and yet distinguishable from it.

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