The philosophical meaning of the Gödel universe

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During his time at the *Institute for Advanced Studies*, Kurt Gödel became a close friend of Albert Einstein, and in particular studied the theory of relativity. One result of this study was the discovery of the so-called *Gödel universe*[1], a model of Einstein's field equations of general relativity in which no absolute time can be defined—theoretically, time travel would be possible in such a universe.

Sometimes, far-reaching philosophical consequences are attributed to this result. Palle Yourgrau, for example, argues that Gödel concluded that there is no time even in our world.[2] In fact, only much more cautious conclusions can be found in Gödel's writings. In this paper, we will first reconstruct the "discovery" of Gödel's universe in the context of his work as a logician.On the basis of this reconstruction, the specifically philosophical challenge that Gödel saw can then be discerned: Our cosmological theories should be capable of being strengthened by means of *physical* principles, so that the nonexistence of Gödel universes—and thus the existence of time—does not depend "on the particular way in which matter and its motion are arranged in the world."[3]

Gödel as a logician

The most outstanding result of Kurt Gödel in the list of his scientific achievements is without doubt the incompleteness theorems [Gö31]. Before that he had already shown the (semantic) completeness of the given axiomatization of predicate logic [Gö30], and he had introduced the recursive functions as a technical tool to prove his incompleteness theorems. Later on, he succeeded to prove the consistency of the Axiom of Choice and the Continuum Hypothesis with the axioms of set theory of Zermelo and Fraenkel [Gö38]. In addition there are many "smaller" results, like a form of the double negation interpretation for Peano Arithmetic in Heyting Arithmetic [Gö33] or the definition of his system of functionals of higher type $\mathcal{T}$, which allows a non-finitist consistency proof of arithmetic [Gö58]. All these works are belonging to mathematical logic, a discipline which has obtained its modern development not least by Gödel's contributions.

It is known that Einstein said towards the end of his life that he only went to the Institute at Princeton to discuss with Gödel.[4] The subject of the discussions

[1] Strictly speaking, it is about a whole class of universes; the singular form stands here as a collective noun.

[2] "Can we really infer the nonexistence of time in this world from its absence from a merely possible universe? In a word, yes. Or so Gödel argues." [You05, p. 130].

[3] Compare [Gö56, p. 412].

[4] Oskar Morgenstern in a letter to Bruno Kriesky, the Austrian Federal Minister of Foreign Affairs, Oct. 25, 1965, [Mor02]: "Einstein often told me that in the last years of his life he always sought Gödel's company in order to be able to discuss with him. Once he said to me

will certainly have been relativity and in particular Gödel's famous Universum, which seems to call our concept of time into question. In the following reflection on the philosophical consequences that Gödel drew from the discovery of his universe, however, it is important to keep in mind that this discovery, too, should be considered against the background of Gödel's *logical* competence.

Einstein's theory of relativity

Albert Einstein had first developed the *special* theory of relativity [Ein05]. It is true that Einstein himself was not a logician. But we can obtain the special theory of relativity absolutely with the help of certain logical guidelines.

The central idea is: the knowledge that there is an absolute speed of light c, which is at the same time the physically highest speed, can be set up as a kind of *axiom*, so that all further theory components of a physical description of our universe must be compatible with this axiom.

This can be illustrated by the addition of velocities. According to classical physics velocities can be added arbitrarily. For example, let a train with the speed v be on the way. If a person in the train goes forward with velocity v, the velocity considered from the outside is $u+v$. Now a passenger holds a flashlight in the direction of travel. The light of this flashlight would then, viewed from outside the train, travel at the speed $u+c$, which would be greater than c, according to the classical view. But if c is to be an absolute maximum speed, the addition must be calculated in a different way. It now requires elaborate mathematical considerations,[5] in order to arrive at the "correct" addition formula of Einstein:

$$\frac{u+v}{1+\frac{u\cdot v}{c^2}}$$

In a simplified description one arrives at the general theory of relativity [Ein16] from the special theory of relativity, if one still considers the further "axiom" that there is *no distinguished frame of reference.* For this one must rework the theory with still far more difficult mathematical tools in such a way that also this axiom is respected in all conclusions.

Viewed through logical glasses, one could say that Einstein developed both the special and the general theory of relativity by *thinking through* new axioms: He reworked the existing theories to the point where they were consistent with these axioms. This, of course, is not to claim an adequate rendering of Einstein's historical approach. It is only about pointing out the specific status of the new assumptions by which one finally obtains a mathematical theory. And with Hilbert such mathematical theories can always be grasped axiomatically [Hil18].

that his own work no longer meant much, that he merely came to the Institute building to have 'the privilege of walking home with Gödel.'"

[5] Note Minkowski's astonishment at Einstein's mathematical achievements, handed down by Max Born:

> Oh, Einstein, he was always skipping lectures—I wouldn't have believed it of him.

From Constance Reid comes the following quote of Minkowski [Rei70, p. 112]:

> Einstein's presentation of his deep theory is mathematically awkward—I can say that because he got his mathematical education in Zurich from me.

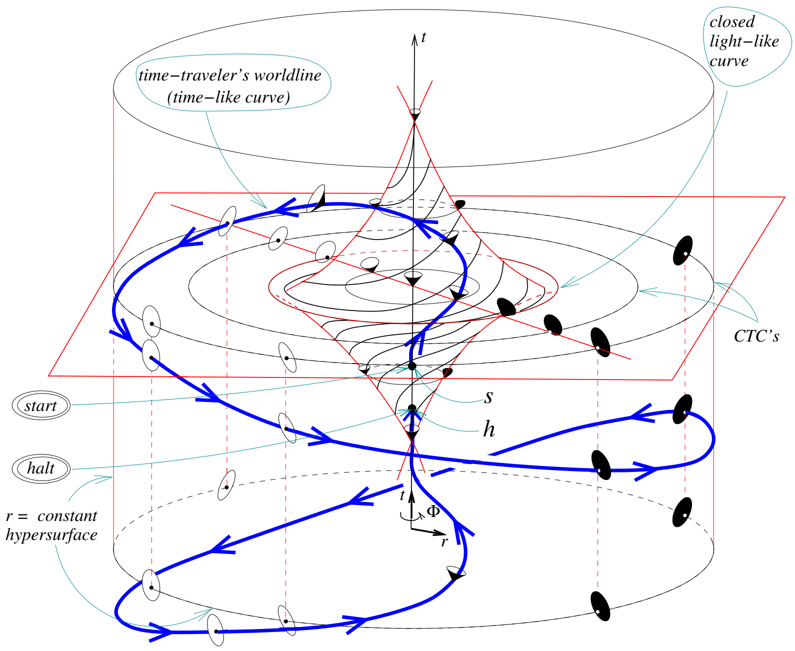

Figure 1: Graphical illustration of a Gödel universe with "time travel" drawn in, taken from [NMAA10].

The Gödel Universe

Gödel could show for the field equations of general relativity that there is a model [Gö49a] in which—theoretically—time travel is possible (see Figure 1).

Even if extensive and complicated mathematical calculations were necessary to prove the existence of this universe, the qualitative conception is a purely logical one: the axiomatic version of the theory of relativity in its field equations *admits non-standard models*; i.e., in particular, that it is formally incomplete.

One could characterize Einstein's procedure to the effect that he investigated what *follows* from the constancy of the speed of light. Gödel then examined in his turn what *is compatible* with the result of Einstein's investigations.

The incompleteness discovered thereby has no specific relation to the incompleteness of arithmetic theories, shown by Gödel in his first incompleteness theorem. This was obtained syntactically and has a generic character, i.e., it carries over to (recursive) extensions of the given theory. Here we are rather dealing with the phenomenon known from absolute geometry (i.e., Euclidean geometry without the parallel axiom). This is incomplete, precisely because the parallel axiom can neither be proved nor disproved, and this has been shown model-theoretically, i.e., by the construction of the non-Euclidean geometries. Correspondingly, the casually formulated sentence: "There is no time travel." has been proved to be independent of the theory of relativity by the construction of the Gödel universe.

Is time travel possible?

The question whether time travel is possible with the discovery of the Gödel universe is badly posed in itself. The modal concept of possibility first requires a definition of the "possibility space", i.e., of those properties and facts which shall be allowed to be changed. This question concerns us in the next section.

But even without a determination of the possibility space Gödel could already deny the posed question. He refers to the time and energy requirement for a time travel which is only theoretically possible in his universe; practically the space-time ship would be too heavy and a journey during which one could visit oneself in the own past would take much too long (respectively would require acceleration which would hardly be acceptable for the human body).

However, Gödel's main argument against time travel is another one: Even if time travel should be possible in "his universe", this does not mean, of course, that "our universe", i.e., the one in which we live—and in which also Gödel lived—is such a universe. Of course, the theory also allows that it is exactly as we and Einstein have imagined it "actually". The Gödel universe possesses a space (time) curvature which would be measurable. In our universe, however, the corresponding red shift cannot be observed. Therefore we live—fortunately—in a "standard universe", in which also the concept of time does not collapse.

The philosophical challenge: Physical principles which exclude the Gödel universe

For Gödel, the actual philosophical challenge lies in the following dilemma: It may be true that "our universe" does not allow time travel, but the fact that the *physical framework* allows other universes in which there is no meaningful concept of time is philosophically more than unsatisfactory. In fact, the existence of time would then depend only "on the particular way in which matter and its motion are arranged in the world" [Gö56, p. 412]. But these are *contingent properties* of our world, which do not already necessarily follow from the physical framework.

Thus, if we take up the question of possibility space raised above, for Gödel the physical principles incorporated in the general theory of relativity established by Einstein—which are expressed, in particular, in the field equations—are firmly given as necessary boundary conditions. But these allow models (in the logical sense) in which "there can be no objective course of time"[6]. If these models can be excluded only by contingent properties—namely the matter distribution and its motion—this can "hardly be regarded as satisfactory" from the philosophical point of view[7].

The task, which Gödel discerns here, consists exactly in finding further *physical* principles, from which the non-existence of his universe would already follow, without the necessity to use contingent properties of our universe.[8]

We have to admit that it is not determined from the beginning what should be considered as a physical principle.[9] The constancy of the speed of light as well as the absence of an excellent reference frame are certainly such principles; the matter distribution and its motion in a concrete universe, however, are not. A trivial solution would be to put the existence of an objective course of time as a principle at the beginning. Such a *petitio principii*, of course, cannot satisfy. Nevertheless exactly this was tried by Stephen Hawking with his "Cronology Protection Conjecture" [Haw92], but just only as a conjecture and not as an axiom. Thus one must credit Hawking that this conjecture is still to be proved by *other* principles.[10]

Until today there is no satisfactory answer to the question of such other physical principles which exclude the Gödel universe. Gödel himself, for example, has considered the *entropy theorem* as a possible candidate. However, he himself has stated that this is also compatible with his universe [Gö56, p. 411].[11]

[6][Gö56, p. 412].

[7][Gö56, p. 412].

[8]With this evaluation we are in contrast to Yourgrau, who writes: "First, he remarks that in an attempt to 'specify' the above definition of 'cosmic' or 'absolute' time, arbitrary elements come into play which can never be fully eliminated" [You02, p. 276, our emphasis]. Here we refer to footnote 9 in [Gö49b]. There, however, Gödel only opposes a precise definition of absolute time in the sense of Jeans, and instead of "never" Gödel carefully speaks of "perhaps", even if he subsequently considers the existence of such a precise definition as *doubtful*. Regardless of any precise definition of absolute time that Jeans may have had in mind, one certainly cannot imply to Gödel that all forms of physical completion of relativity could be accomplished only by arbitrary elements. This follows already from his own considerations, which will be briefly addressed below.

[9]Gödel says this explicitly at the end of the quotation given in footnote 12.

[10]From this perspective, Yourgrau's harsh criticism of Hawking [You05, pp. 8 and 136] is both incomprehensible and unjustified.

[11]This remark can be found as an "addition by the author in the German translation to

Gödel has also thrown a particularly interesting spotlight on the question raised about the distinction between physical principles and contingent properties:[12]

> What used to be a practical difficulty in microphysics has now, as a result of the indeterminacy relation, become an impossibility in principle; and the same may one day occur for the difficulties based not on a "too small" but on a "too large".

Thus, if quantum mechanics invalidates the (by now) known laws of relativity at (too) small scales, Gödel speculates on the possibility of a "cosmological physics" which, for its part, would revise the known laws at (too) large scales.[13] To our knowledge, however, this idea has not been pursued further to this day.

Finally, we want to emphasize that Gödel carries out the philosophical speculation about the existence of time in a specific context, namely in what he calls *idealist philosophy.*[14] This is prominently referred to in the title of the work [Gö56], and he writes, with regard to the problems to which the nature of time is exposed by relativity: "It seems [...] that one obtains a clear proof of the view of those philosophers who, like Parmenides, Kant, and the modern idealists, deny the objectivity of change and regard it as an illusion or as an appearance which we owe to our particular mode of perception." On closer examination of Gödel's argumentation in this article, one can see that he does not primarily support the (only apparent?) "clear proof". He merely argues against the fact that such a proof can already be brought down by the fact that one can save the existence of time by recourse to the *contingent* properties of our universe.[15] Although he does not comment on whether he still considers a non-idealistic philosophy to be viable as an alternative;[16] he clearly looks after the possibility of guaranteeing the existence of time through deeper physical principles.

To find such principles is the philosophical challenge that Gödel has left us.

footnote 14" only in the German edition of the article.

[12]This can be found again in an "addition of the author to footnote 11 of the German edition" which is introduced as follows [Gö56, p. 411]:

> A second reason to exclude the above universes a priori could be found in the possibility of a "telegraphing into one's own past". But the practical difficulties arising in this case are hardly likely to be less [than in the case of the previously discussed time travels]. By the way, the border between practical and principal difficulties is by no means immovable.

[13]Here one may well feel reminded of the Aristotelian distinction between sublunary physics and celestial mechanics.

[14]The terminology used by Gödel is not unproblematic. In which concrete sense the presented position should correctly be called *idealistic* may be discussed. The tension is increased by the fact that Gödel himself is usually regarded as a realist; see e.g. [You05, p. 171f.].

[15]He explicitly attributes such a rescue attempt to James Jeans in [Jea35], [Gö56, p. 408].

[16]Howard Stein [Ste90], in his introduction to the reprint of the article [Gö49b], explicitly regrets the lack of more material that would shed light on Gödel's philosophical position, especially vis-à-vis Kant. We have at least more detailed drafts of this article, in which "idealistic philosophy" was replaced by "Kantian philosophy" in the title, and which are dated 1946–49 [Gö95].

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