Monads, Types, and Branching Time – Kurt Gödel's approach towards a theory of the soul

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January 6, 2020

Abstract

Three unpublished notebooks on the foundations of quantum mechanics, written by Kurt Gödel in Vienna, clearly show that Gödel studied Leibniz' Monadology in depth as early as 1935, and regarded it as a metaphysical frame for quantum mechanics and physics in general.

Drawing upon original source material, this paper presents two different links between the concepts of *Monad* and *worldview*, the first one being based on a type-theoretical analogy between the universal set and an objective thing-in-itself, the second one involving the idea of branching time, represented by a tree-like partial order on the set of viewpoints. Following Leibniz' remarks closely, both links contribute to a metaphysical theory of the soul, which Gödel was persuing at the time.

1 Introduction

In 1935, Kurt Gödel wrote two consecutive notebooks entitled '*Physik – Quantenmechanik I*' and '*Physik – Quantenmechanik II*'¹, which—to this day remain completely unpublished but have now been entirely transcribed from the Gabelsberger shorthand system by the present author. The books contain Gödel's thoughts, ideas, and questions about the foundations of quantum mechanics, carefully devised into a single list of about 340 items. Only one year later, Gödel wrote a third notebook², entitled '*Aflenz 1936 (Analysis, Physik)*', compiled on the basis of the earlier two books. Whereas some of the items are simply just copied, others are carefully revised and sometimes extended. Also, many of the original notes are completely dropped, with the original order of the items being retained.

In a reply to a questionnaire sent to Gödel by Burke D. Grandjean in 1974, published by Hao Wang in (*Wang*, 1987), Gödel himself explains that he studied Leibniz between 1943 and 1946, adding that 'the greatest phil. infl. on $\langle \text{him} \rangle$ came from Leibniz.' Gödel's books on quantum mechanics now clearly prove

¹Henceforth called QM1 and QM2, respectively. Kurt Gödel Papers, Box 6b, Folder 78, item accessions 030106 and 030107, on deposit with the Manuscripts Division, Department of Rare Books and Special Collections, Princeton University Library. Used with permission of Institute for Advanced Study. Unpublished Copyright Institute for Advanced Study. All rights reserved.

²Kurt Gödel Papers, Box 6a, Folder 59, item accession 030082.

that he did indeed study at least Leibniz' Monadology³ as early as 1935 (or even earlier). Side by side with rather technical considerations, the books also contain many philosophically orientated comments, in many cases closely connected to Leibniz' Monadology.⁴

The following section will concentrate on an analogy between the *universal* set and an objective thing-in-itself. Here, Gödel describes what might be seen as a kind of monadologic type theory. Section 4 then concentrates on a more complex link between monads and worldviews, with the latter being organized in tree-like structures, representing models of branching time. Section 5 briefly reviews the history of branching time, comparing Gödel's notion to those introduced by Saul Kripke (in connection with A.N. Prior's tense logic) and Nuel Belnap (in connection with relativity and indeterminism).

Throughout the paper, Gödel's notes are presented as close to the original as possible. [Square brackets] and (parentheses) are Gödel's own, further additions by the present author are marked in (angle brackets). Although the items in the later Aflenz book are often more elaborate, the particular case always decides, which comment suits the situation best. Also note that the last item of QM2, which has been considered in the Aflenz book, is item 318. As the order of Gödel's notes (and sometimes even within the notes) does not necessarily reflect his overall train of thought, this paper is aiming at reconstructing and presenting Gödel's main ideas. For brevity, we will concentrate on the central passages of the items in question.

2 Monads and types

The application of Leibniz' monadology to quantum mechanics and physics in general starts with a first vague comment on metaphysical systems regarded as 'frames for physical theories'. In item 266 of QM2 Gödel writes:

Die metaphysischen Systeme sind nichts anderes als verschiedene Rahmen für physikalische Theorien. [Wie sieht das Platonische aus?] Bisher wurde immer nur das Demokrit'sche System als Rahmen in der wirklichen Physik verwendet. In der modernen Physik wird es anders werden. Vielleicht das Leibniz'sche an seine Stelle?

As we shall see, the Leibnizian system in question clearly is his monadology, offering a neat possibility to approximate an objective physical reality and closely linking monads with the idea of different worldviews. The connection between monads and worldviews is describes in item 250 of QM2.

Vielleicht ist $\langle die \rangle$ Leibniz'sche Monadolgie eine der Zwischenstufen zwischen Solips. und objektiver Theorie. Charakter: Man hat nicht "ein" "wahres" Weltbild, sondern ebenso viele verschiedene Weltbilder, als es Monaden gibt.

The close connection between monads and worldviews is of course no stranger to the Monadology. Here Leibniz states⁵ (\S 57): 'And as the same town, looked

 $^{^{3}}$ Item 217 in QM1 also mentions Leibniz' dissertation 'De principio individui'. It reads: 'Der Raum verletzt nach Leibniz das Princ. Id. und daher etwas Ideales. Raum = Princ. Individuationis'

⁴Other philosophical notes concentrate on neovitalistic concepts.

⁵Throughout this paper, we use Robert Latta's 1898 English translation of the Monadology.

at from various sides, appears quite different and becomes as it were numerous in aspects [*perspectivement*]; even so, as a result of the infinite number of simple substances, it is as if there were so many different universes, which, nevertheless are nothing but aspects [*perspectives*] of a single universe, according to the special point of view of each Monad.'

In the aforementionend item 250, Gödel then begins his detailed description of an approximation of an objective theory by pointing at an analogy between the *universal set* and an objective *thing-in-itself*. Whereas type theory can be seen as an approximation of the (non-existent) contradictory universal set, an infinite layering of monadologic structures is able to approximate an objective thing-in-itself as well as an (non-existing) objective physical theory. In his Aflenz book, item 250 begins with the following table:

Analogie zwischen "Allmenge" und "Ding-an-sich":

Russell Antin.	Planck Antin.
In keiner mathematischen The-	In keiner Annäherung an die
orie kommt die Allmenge vor.	Wirklichkeit kommt ein vollkom-
(Nur in der widerspruchsvollen	men "objektives" Ding-an-sich
alten.)	vor. (Außer in der wider-
	spruchsvollen klassischen.)

The role of a theory of types is maybe best described in the corresponding item in QM2:

Hauptproblem: Für die physikalische Theorie dasselbe leisten, was die Typentheorie für die Logik geleistet hat. D.h., welches ist die <u>Struktur</u> derjenigen⁶ Folge von Theorien, welche die klassische Theorie einer "objektiven Welt" zu ersetzen und zu approxim. hat? Ein wesentlicher Bestandteil dabei muss offenbar die "atomistische" (monadologische) Struktur der Welt (sein).

Before we have a closer look at the layers of typed monadologies, it is worth considering Gödel's concept of solipsism. As described in item 250, monadology is seen as an intermediate stage between solipsism and an objective theory. In this respect, solipsism represents an isolated and highly subjective worldview, which does not accept—or rather ignores—the existence of any other viewpoints. The way to overcome this isolation (and at the same time the step into a monadologic world) is the process of an approval of the existence of other entities, i.e. of other monads. In item 254 of the Aflenz book, Gödel very briefly mentions this idea as:

Monadologie = Solips. + Anerkennung des Du

Here, the main constituent is indeed the 'acceptance' or 'approval' ('Anerkennung') of other monads, i.e. of other worldviews, which do exist in different (coexisting) solipsistic worlds. This change of perspective is clearly expressed in item 265 of QM2:

Quantenmechanik: Einfachste Beschreibung der Welt ist nicht mehr als objektive Welt der Dinge (invariante Beschreibung), sondern Beschreibung der verschiedenen Solips. Welten + Transform. Gesetze.

⁶Gödel's footnote: transfiniten

Once an isolated solipsism has been overcome, one ends up with a first layer consisting of an infinite set of monads. This very first layer can be characterized by the absense of the Leibnizian concept of a '*universal harmony*' (§59), which interconnects the monads. Nevertheless, the simple substances do comprise an inner activity, as Leibniz describes in §18: 'All simple substances [...] have a certain self-sufficiency which make them the sources of their internal activities and, so to speak, incorporeal automata.' A first level of *harmony* is given by Gödel again in item 250 of QM2:

 $\langle \mathrm{Die} \rangle$ Leibniz'sche Monadologie ist als Zwischending zwischen Solips. und objektiver Welt nicht willkürlich, denn: $\langle \mathrm{Der} \rangle$ Grund, weswegen man bei Solips. nicht stehen bleiben kann, ist das Du. D.h., es bestehen gleichberechtigte "Selbste", zwischen denen dann offenbar Abhängigkeiten bestehen müssen. [Das ist das Schema (Struktur) der Monadentheorie.] (Die Art dieser Abhängigkeiten kann am besten beschrieben werden durch Spiegelung, aber nicht Spiegelung des Universums, sondern bloß Spiegelung seiner selbst durch tausende von Spiegeln.)

Here Gödel follows exactly Leibniz' description of a *universal harmony* amongst the infinite set of monads or simple substances, at least as far as the mirroring relation *between* the monads is concerned. In §56 of the Monadology Leibniz writes: 'Now this connexion or adaptation of all created things to each and of each to all, means that each simple substance has relations which express all the others, and, consequently, that it is a perpetual mirror of the universe.'

Approximating an objective physical theory by means of a kind of monadologic type theory, Gödel certainly has to reject Leibniz' 'consequence', a mirroring of the entire universe. Also, the actual mirroring is an iterated process and takes place in distinct layers of monadologic theories, thus paralleling the layers of set theoretic types. In item 250b of the Aflenz book, Gödel describes this iteration as follows:

Zum Prinzip, welches von T zu T' führt: T_0 , es sind nur die Monaden da (ohne Inhalt, d.h. Vorstellungen). T_1 , jede Monade enthält das Bild der übrigen (leeren) Monaden. T_2 , jede Monade enthält das Bild der übrigen Monaden mit den Vorstellung aus T_1 , usw.

At this point, the similarity to set-theoretic types becomes evident: The empty monads resemble 'empty' objects, for instance natural numbers. On the next layer the mirroring monads resemble sets of naturals numbers, and on the following layer they resemble sets of sets of natural numbers, and so on.

Summing up, Gödel clearly underlines a strong parallel between a logical and a monadologic type theory, both constituting an infinite approximation of something unreal and contradictory, the universal set in one case, an objective thing-in-itself in the other. In both cases, a necessary component is an initial step from a single isolated entity to an infinite set of objects. The overall situation is nicely summarized by Gödel himself in item 248 of his Aflenz notes:

Bei konsequenter Durchführung der pos. Quantenmechanik muss irgendwo das intersubjektive Moment hineinkommen (Loskommen vom Solipsismus). D.h., die unendlich vielen Gesichtspunkte (subjektive Weltbilder) sind gesetzmäßig verknüpft, ohne dass diese gesetzmäßige Verknüpftheit auf ein gemeinsames "Ding-an-sich" zurückgeführt wird⁷ (= Unterschied

⁷Gödel's footnote: oder Ding-an-sich sehr eigenschaftsarm

gegenüber alter Physik). Hier transfinites Moment, da die Abbildung selbst wieder abgebildet wird.

3 Branching time and a theory of the soul

Up to now, each Monad comprises exactly one point of view, thus establishing a clear one-to-one relation between monads and viewpoints. In a subsequent step, Gödel emphasises a relation between a special kind of Monad and a treelike structure of viewpoints, the 'person'. His basic idea is again based on the Monadology. In §19, Leibniz writes: 'If we are to give the name of Soul to everything which has perceptions and desires [appetits] in the general sense which I have explained, then all simple substances or created Monads might be called souls; but as feeling [le sentiment] is something more than bare perception, I think it right that the general name of Monads or Entelechies should suffice for simple substances which have perception only, and that the name of Souls should be given only to those in which perception is more distinct, and is accompanied by memory (emphasis added).' And he continues in §26: 'Memory provides the soul with a kind of consecutiveness (emphasis added), which resembles [imite] reason, but which is to be distinguished from it.' Gödel himself states in item 273 (QM2):

Bewußtsein ist nur möglich durch Mneme.

And it is interesting to note that he repeatedly askes the question if a very simple and primitive form of life, namely the *paramecium*, does indeed have a memory.⁸

In a subsequent step, Gödel now connects the concepts memory and viewpoint. In item 322 of QM2, he obviously describes a mathematical structure $\mathfrak{V} = (\mathcal{V}, \mathcal{P}, \mu, \rightarrow, d)$, where \mathcal{V} is the infinite set of viewpoints, and $\mathcal{P} \subset \mathcal{V}$ is the set of so-called possible viewpoints. If \mathcal{M} is the set of monads, $\mu: \mathcal{V} \rightarrow \mathcal{M}$ maps each viewpoint into the set of monads. \rightarrow is a partial order on the set \mathcal{V} of viewpoints, stipulating that $A \rightarrow B$ implies $\mu(A) = \mu(B)$, interpreted as 'linked by memory', both viewpoints A and B belonging to the same Monad. d denotes a metric on \mathcal{V} , interpreted as the 'distance' between two viewpoints. Gödel himself describes this structure \mathfrak{V} in item 322 (QM2) as follows:

Eine "pos." Theorie der Wirklichkeit hat wahrscheinlich folgende Struktur: Grundelemente: die unendlich vielen "Gesichtspunkte". Diese zerfallen in wirkliche Gesichtspunkte und mögliche Gesichtspunkte. [Die ersten sind solche, in denen sich tatsächlich eine Monade befindet.]

- I Zwischen den verschiedenen wirklichen Gesichtspunkten bestehen Beziehungen.
 - a.) Die Gesichtspunkte A und B sind [in der Richtung $A \to B$] durch Erinnerung verbunden, d.h. insbesondere, sie gehören derselben Monade an.
 - b.) Die Gesichtspunkte A, B sind nahe bzw. sind fern voneinander. [D.h., das Bild, welches die Welt für sie bietet, ist mehr oder weniger ähnlich.], etc.

⁸In item 263 (QM2), repeated in the Aflenz book, Gödel writes: 'Behauptung, dass auch ein *Paramäzium* (sic) (ein) Gedächtnis habe! (Nat. 1934, Bleuler, nachsehen Semon).' And in a crossed out item 274 (QM2) he askes: 'Ist das Gedächtnis des Paramäz. (sic) wahr?'

Reading on, it becomes clear that the aforementioned basic structure \mathfrak{V} obviously has to be enhanced by a system F of functions which—for each viewpoint—produce an 'image of the world', as Gödel calls it. Finally, these images are connected by physical laws if their underlying viewpoints are related by the partial order \rightarrow , the smaller the distance d between two viewpoints, the more similar their images.

- II Jedem Gesichtspunkt ist zwischengeordnet ein "Bild der Welt", dargestellt durch ein gewisses Funktionensystem.
- III Die Axiome der Physik sagen aus, dass zwischen zu verschiedenen Gesichtspunkten gehörigen "Bildern" verschiedene Beziehungen bestehen, falls diese Gesichtspunkte in bestimmter Weise durch Relationen I verknüpft sind.

The very last item of QM2 (340) now explains Gödel's concept of a 'person' and interconnects it with viewpoints, monads, the notion of free will, and the phenomenon of time. Under the heading 'Bedeutung des freien Willens und Möglichkeit seiner widerspruchsfreien Vereinigung mit Det.', Gödel writes:

Zunächst ist eine Person nur eine Menge von Gesichtspunkten. Wieso bekommt diese Menge eine Struktur (Ordnung)? Hängt zusammen mit Anm. 2, dass ein Gesichtspunkt A irgendwie als "Objekt" eines anderen Gesichtspunktes auftreten kann: B später als A, wenn A Objekt von B, aber nicht umgekehrt.

Das Phänomen der Zeit besteht

- 1. darin, dass eine mögliche Person nicht ein Gesichtspunkt, sondern eine Menge von solchen ist,
- 2. dass die Relation des "Objekt-Seins" asym. ist, oder wenigstens besteht eine Person nur aus solchen Gesichtspunkten, für welche das zutrifft, und zwar aus einer möglichst großen Menge. Dies (ist) übrigens vielleicht nur Charakteristikum derjenigen Personen, welche unserer Beobachtung zugänglich sind. [Vielleicht gibt es andere Existenzformen, z.B. mit zweidimensionaler Zeit, etc.]

Earlier in the same item, Gödel describes his idea of the possibility of a person's influence on the natural flow of time within a branching scheme of viewpoints.

Das Fortschreiten der Zeit besteht darin, dass sich der Gesichtspunkt ändert. Der Gesichtspunkt ist teilweise von außen (Schicksal) bestimmt, teilweise (in sehr geringem Grad) durch meinen Willen. Insofern er durch Schicksal bestimmt ist, haftet ihm das Attribut der "Rätselhaftigkeit und des Geheimnisvollen" an. Durch Training $\langle ist \rangle$ eine größere Abhängigkeit vom Willen erreichbar (Fakirismus). [Schon durch Technik: leichtere örtliche Verlegung des Gesichtspunktes.] Im Allgemeinen ist die Folge der möglichen Gesichtspunkte [zwischen denen ich zu wählen habe] durch ein Verzweigungsschema in der Richtung wachsender Zeit gegeben. Erinnerung könnte aufgefasst werden als eine Möglichkeit, den Gesichtspunkt aus diesem Schema heraus in die Vergangenheit zu verlegen.

Figure 1 reflects the overall situation, focusing on a set of viewpoints belonging to the same Monad.

Finally, the difference between 'real' ('wirklichen') and 'possible' ('möglichen') viewpoints is explained—at least to some degree—by sharpening the notion of 'person'. Again in item 340 (QM2), we get the following piece of information:

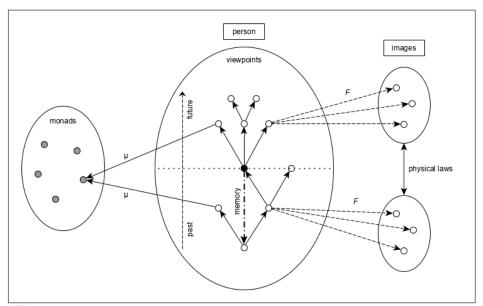


Figure 1: Gödel's theory of the soul in a nutshell. Each person comprises a tree-like structure of viewpoints which all belong to the same Monad. This very Monad rests in one of the viewpoints (marked in black) right between past and future. Memory is regarded as the possibility to change the Monad's viewpoint into the past. Each viewpoint has an 'image of the world', and if two viewpoints are transitively linked by the relation \rightarrow , the corresponding images are connected by physical laws. (Not pictured is the metric d.)

Beim Begriff einer Person ist zu unterscheiden zwischen "möglicher" Person und "wirklicher Person". Nicht je zwei mögliche Personen "passen zusammen". Die Menge der wirklichen Personen muss eine Menge von "zueinander passenden" Personen sein. [Dies ist das Wechselwirkungsproblem der Teilchen.] D.h., die Frage, ob eine Menge eine mögliche Person ist, ist so zu verstehen, ob sie ergänzt werden kann zu einer vollständigen möglichen Menge von Personen. Je nach dem, ob die Ergänzung auf viele oder wenige Arten möglich ist, wird man von einer Wahrscheinlichkeit sprechen.

4 The history of branching time revisited

In 1950, Arthur N. Prior (1914–1969), well known for his founding work in modern temporal logic, had become interested in the logical studies of the Megarian logician Diodorus. Based on Diodorus ideas concerning time and possibility, Prior represented propositions as infinite sequences of truth values, reflecting the present as well as the future development of that very proposition in terms of truth. If p now is a proposition in this sense, the proposition $\Diamond p$ (again an infinite sequence) has to be interpreted as follows: An element of $\Diamond p$ is true, if an element either at the corresponding or any 'later' position in p is true. In (*Prior*, 1957), Prior then erroneously specified **S4** to be the underlying logical system. Very soon, Saul Kripke, who had discovered the error⁹, wrote a letter to Prior which in (\emptyset hrstrøm, 2010) is described as 'one of the most important events in the history of logic during the 20th century.' Kripke writes (*Kripke*, 1958): 'Now in an indetermined system, we perhaps should not regard time as a linear series, as you have done. Given the present moment, there are several possibilities for what the next moment may be like — and for each possible next moment, there are several possibilities for the next moment after that. Thus the situation takes the form, not of a linear sequence, but of a 'tree'.' At the same time, Kripke was able to show that these models of branching time indeed satisfy the logic **S4**. Following (\emptyset hrstrøm, 2010), this was the very first appearence of the idea of branching time models.

Many years later, in 1992, the American philosopher and logician Nuel Belnap, who had been in close contact to Prior in the 1950s, carried Kripke's idea over to physics, turning *branching time* into a relativistic spatio-temporal variant, *branching space-time (BST)*, as an attempt 'to do metaphysics in a mathematically rigorous way, with the desideratum: be compatible with current physical theories' (*Placek & Belnap*, 2012). Whereas traditional physicists regarded the flow of time as a linear temporal order on the infinite set of Euclidean three-dimensional spaces, Belnap now described a tree-like 'causal order' on the Minkowski space of four-dimensional 'possible point events', thus merging indeterminism and relativity. As a central disadvantage, Belnap regarded his theory as 'remote from real physics' (*Belnap*, 1992) and later added that it had '(for better or worse) no concept of laws of nature, although it is laws-friendly, since it has modalities and propositions, both rigorously defined' (*Placek & Belnap*, 2012).

Unknown until very recently, Kurt Gödel must have had his idea of branching time already in or even before 1935, immediately integrating it into his theory of the soul. Although Leibniz conception of the soul as a Monad with a memory could have been realized incorporating linear time, Gödel combined it with his thoughts about free will and indeterminism, notions Belnap considered nearly sixty years later. It should therefore be well worth repeating Gödel's headline for item 340 of QM2, in which he presents the concept of branching time: 'Bedeutung des freien Willens und Möglichkeit seiner widerspruchsfreien Vereinigung mit Det.'

The major differences between Gödel's and Belnap's conceptions are twofold: Firstly, Gödel's primitives are viewpoints and a relation consisting of 'links by memory', whereas Belnap's corresponding primitives are possible point events and a causal ordering relation, respectively. The second difference concerns the underlying topology. Whereas in Belnap's theory the entire set of point events is connected by the causal order, Gödel regards a non-trivial clustering of viewpoints, with certain tree-like clusters, the 'persons', playing a special role, their viewpoints being connected to the same Monad. Furthermore, Gödel's firm connection between viewpoints and physical images allows for a smooth transition from metaphysical structures to physics itself.

⁹Kripke had noted that $\Box \Diamond p \lor \Box \Diamond \neg p$ was valid in Prior's system but could not be deduced in **S4**.

5 Conclusion

When Oscar Morgenstern noted in his diary (August 28, 1970), that Gödel had stated that his ontological proof for the existence of God was nothing but a logical investigation, he certainly did not see the entire heavy-weighted background. Gödel worked on his ontological proof at least from 1940 on and kept revising it until 1970, when he finally stated that he was satisfied with his latest variant¹⁰. The motivation for this kind of metaphysical work is precisely expressed in his notebook *MaxPhil IX*¹¹ (pp. 78), where Gödel calls it his main goal in philosophy to define and axiomatize the uppermost philosophical concepts and deduce conclusions and theorems with mathematical rigidity. And in a conversation with Rudolf Carnap in 1940, published in (*Gierer*, 1997), Gödel says: 'Man könnte $\langle ein \rangle$ exaktes Postulatensystem aufstellen mit solchen Begriffen, die gewöhnlich für metaphysisch gehalten werden: "Gott", "Seele", "Ideen". Wenn das exakt gemacht würde, wäre nichts dagegen einzuwenden.' Carnap himself then mentions the analogy to theoretical physics. All these facts clearly underline the essence of Gödel's metaphysical program.

The now discovered and transcribed notes concerning the formalization and axiomatisation of the concept of 'soul' on the basis of Leibniz' Monadology therefore contribute to a deeper understanding of Gödel's metaphysical plan, about which Hao Wang wrote in 1987 (*Wang*, 1987, p.29): 'But G appears to have sometimes aimed for at an even higher level of achievement: doing for metaphysics what Newton did for physics $\langle ... \rangle$. I am not able to determine how far G progressed toward such ambitious goals or what evidence he had for believing them attainable.' The notes also show that, although Gödel never published anything about his idea of branching time, he foresaw all the main features which branching space-time brought into physics, nearly sixty years before this theory was finally published.

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 $^{^{10}}$ For the 1970 variant, see (*Gödel*, 1970). For an exposition of Gödel's different variants and their development, see (*Kanckos & Lethen*, 2019).

¹¹Kurt Gödel Papers, Box 6b, Folder 69, item accession 030095.

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