Gödel's Incompleteness as an Expression of the Fragmentation of Epistemology in Mathematics and the Science

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1 Introduction

Does incompleteness in Gödel's sense play a role in the relationship between micro- and macrophysics, and if so, what consequences would that have?

The incompleteness theorems published by Kurt Gödel in 1931 sent a ripple through the then ongoing debates concerning the foundations of mathematics. It seemed as though all of those factions in the crisis of mathematics which strove to establish a logically coherent common basis for mathematics were bound to fail. Whether it was purported that the field needed to be derived from the logical structure of syntax, or instead from axioms of a most precise formalism, they would always carry the burden of incompleteness with them. Since then, the set theory of Zermelo and Fraenkel has been widely accepted as the foundational system from which all other fields of mathematics may be derived, given one supplements it with the axiom of choice. The crucial difference to Cantor's original naive set theory lies in the circumstance that we can now be certain that there is no certainty in these axioms. They appear to be consistent and provide for a solid basis, however, depending on the development of mathematics, they may as well need to be tweaked or even replaced. This way, a form of falsificationism appears to have entered metamathematics. Just like in the natural sciences, mathematics entails a form of debunking and demonstration which runs analogously to experimental science. Gödel also has indicated a similar view in his arguments against Rudolf Carnap (Gödel, 1953). Arguably, scientific theories, especially those of theoretical physics represent formal systems. This means that the same implications which the incompleteness theorems had on mathematics can be found in science as well. There are explicit demonstrations of this, e.g. the undecidability of the spectral-gap problem (Cubitt et al., 2015). However, the phenomenon is observable throughout the sciences and arguably represents a general philosophical issue.

2 Epistemic Fragmentation

First of all, it is important to note that other than the foundational crisis of mathematics, Edmund Husserl had also observed a *crisis of European sciences* (Husserl, 1936) occurring over the course of the scientific revolution. Before the enlightenment, science derived its unity from the fact it was subject to the government of the prime philosophy of metaphysics. The system of scholasticism had united Christian theology with the philosophical conceptualisation of the absolute. In the same way, the Platonism of Cantor attempted to generate all mathematical elements through their subordination to the absolute, conceptualised as the set of all sets. The mathematical issue was partially resolved by doing away with the strict need of unity and introducing the class of sets instead. Earlier, in the philosophy of science, it had come to the *Cartesian split*, in which the absolute was split into a dualism of *res extensia* and *res cogitans*, two irreconcilable substances. The attempt at postulating a unifying bridge, namely God, as that which holds the polarities together, failed and lead to an increasing polarisation of the dichotomy (nowadays observable as the distinction between humanities and sciences).

The preceding elaboration, however, does not explain the original reason for the disintegration of metaphysics. The answer can be found in the relationship between metaphysics and logic. Gotthard Günther observed a form of asymmetry within metaphysics which lays at the heart of our understanding of logical consistency and incoherence (Günther, 1957). Günther rejected the contemporary notion that it is somehow possible to apply formal logic without recourse to the Aristotelian metaphysic within which it emerged. The asymmetry arises from the establishment of a form of duality within the classical system in which there was an eternally true being and a potentially true perception of being (Aristotle). These two poles were not a balanced dichotomy, as being was superordinate to the perception of it, in the same way as truth and falsehood are applied as equal modalities, whilst not being equally balanced. Truth is superordinate to falsehood, which replicates itself in the liars paradox. The statement "*this sentence is true*" is consistent and self-evidently true, whereas "*this sentence is false*" is not false, but actually inconsistent. This stems from the meaning conveyed in the modalities *true* and *false* which originates from classical metaphysics.

The asymmetrical structure of metaphysics arguably has played itself out throughout many historical dichotomies, such as the prioritisation of being over becoming, the one over the many, or spirit over matter. Similarly a fundamental dichotomy plays itself out within mathematics, demonstrated in its earliest form by Zeno's paradox. This duality is often described as the opposition of the discrete and the continuous. In mathematics, the interplay of these opposites has lead to many developments within the field. The *One* (or the absolute) is something that is somewhat both, continuous and discrete, however, it was classically regarded as discrete, as the existence of irrational numbers on the continuum was long denied. In the enlightenment era, the introduction of real numbers had the natural and rational (the discrete) numbers as a subset of the continuum. However, a suspicion of actual infinities still prevails and computability necessitates an application of discrete measures. Similarly, theoretical physics favours the discrete. General relativity and field theories in general can be regarded as continuous theories, whereas quantum mechanics is a prime example for a discrete theory. The rift fractally replicates itself within the theories, where quantum mechanics entails the infamous problem of the reduction of the wave function. The discrete understanding of the particle during its measurement is privileged as it is seen that information and strict determinism is lost due to the duality of wave and particle. The wave being the continuous form of matter, only ceases to behave deterministically, when trying to regard the wave as a non-actualised form of the actualised discrete particle.

Günther argued that the concept of information was needed to be added as a third realm in addition to spirit and matter (Günther, 1957). An information theoretical approach can be applied to spiritual and material entities alike. It has been seen in theoretical physics that the analysis of statistical thermodynamic information on black hole surfaces enabled the bridging of quantum theory and general relativity in certain areas. Within that framework, gravity can be regarded as an emergent property of a system approaching maximum entropy (Verlinde, 2011). The 'jump' that a system makes when a new emergent property arises cannot be reduced to the components (or 'parts') of the system, as the study of complex systems suggests (Kahle, 2009). It is much rather a property that arises through the synergistic interplay of all parts, so that the system needs to be embedded into a superordinate structure in order to reconcile the system and its parts. The circumstance that Gödel's proof is constructed by means of a self-reference, which is undecidable within the system, but decidable from a superordinate system, is a symptom of such an emergent property.

Arguably, the incompleteness of formal logic takes its origin from the fundamental irreducibility of the continuous to the discrete (as Gödel's formal systems need to be recursively enumerable) and thus replicates itself in all such theoretical cases, where it is attempted. The asymmetry of formal logic, exemplified in the liar's paradox, is mirrored in the proof for the first incompleteness theorem. Only does Gödel shift the asymmetry towards a new one. Instead of the difference between truth and falsehood, he focuses on the potentially true provability, and unprovability, which utterly lacks the potential to be true or false. This shift towards provability embeds the notions of truth and falsehood within a broader (perhaps *dialetheic*) context (Priest, 1998). It corresponds with his philosophical shift towards a sense of truth that goes beyond logical modalities.

3 Gödel Rediscovers Ontology

In order to understand the significance of Gödel to the foundations of mathematics and science, it is crucial to emphasize on his relationship to the absolute. Besides speaking favourably of religion (Wang, 1988) and formulating a proof for god (Gödel, 1970), he made a statement, where he expressed joy about having found out something about the absolute by logical reasoning alone. Cantor's antinomies show that the absolute itself represents a contradiction, however, instead of concluding the non-existence of the absolute, Gödel seems to imply a form of negative theology towards it. It exists, however, we cannot speak of it in logical terms. Similarly, his ontological realism of mathematical objects does not regard triangles and numbers to be existing in some off-worldly realm (as Russel would have accused him of), but he regards these objects to be formalisations based on qualitative mathematical perceptions of real entities (a form of intuition or apprehension of *Gestalt*). Therefore the inconsistencies occurring in mathematics represent limitations in the formulation of axioms, similar to the development of scientific theories. He references Husserl (Gödel, 1953) and sees the future of philosophy of mathematics in the rediscovery of Platonism by means of the phenomenological method.

Husserl's student Martin Heidegger was driven at reconciling the pervasive ontological rift in philosophy by means of what he called *Verwindung* (instead of *Überwindung*), a form of overcoming that did not involve the super-ordination of one principle over the other (even not temporarily, as can be found in a dialectic approach). Instead, the dichotomies were said to twist out of their confused entanglement. The focus is now shifted from the ontological rift towards an *ontological distinction* between Being and beings (*Sein* and *Seiendes*). This distinction represents the openness within which Being discloses itself. That being which witnesses said disclosure is *Dasein*, that entity which relates to its own Being (Heidegger, 1927). The relevant aspect of Dasein is that it represents the primordial ground from which the separation of subject and object, and, in effect, the separation of the discrete and continuous, originate. Gödel has proven something of which the consequences are so fundamental that only a few, such as Heidegger, also have hinted at the same fundamental philosophical transitions that are becoming increasingly necessary.

The rediscovery of ontology after its disintegration is a essential necessity for any pursuit of truth. The understanding of truth itself, however changes once the classical metaphysical relationship between entities is dissolved. Gödel has hinted at a direction which has yet to be explored. The alternative is the continuous progression of scientific research towards data science, cybernetics and statistical evaluation. The dissolution of meaning and grand narratives in science essentially renders the search for grand unifying theories a defunct and seemingly outdated project. A project that struggles to rediscover the firm ground it so much desires to stand on.

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